Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam

Spring 2008

PART I

May 26, 2008

Please answer all 4 questions. Notice the time allotted to each question.

Problem 1 (30 minutes)

Virginia Tech has developed a new type of composite material and has a patent on it. Since it does not want to be involved in its production it plans to allow only a single firm, CompMat Inc., to produce the new material. Va Tech knows that the demand in the market for this material is given by

$$q = 10000p^{-2}$$

where q is quantity and p is price. Furthermore, the university knows that CompMat Inc. will not have fixed cost related to the production of this material and that its marginal cost will be

$$MC(q) = 10.$$

The university is considering two different types of contracts with CompMat Inc.. Under the first it will sell the innovation to the firm for a one-time payment of T dollars. The firm will then own the innovation. Alternatively, the university will charge a license fee of αR where R is the revenue that the firm will have and $0 < \alpha < 1$. For reasons of convenience assume that all sales take place during the current period.

(a) If the university decides to sell the innovation, what is the optimal level of T ?

(b) If the university decides to license the innovation, what is the optimal level of α ?

(c) Should Virginia Tech sell the innovation or should it license it?

Problem 2 (60 minutes)

We consider the economy of Terra Nova with one firm and $k+m+n$ consumers, where $k > 0$, $m > 0,\ n > 0$ and three commodities.

THE FIRM: Commodity 1 serves as input (factor of production) for the firm. Commodity 3 is produced by the firm. The firm's technology is described by a twice differentiable production function $f : \mathbf{R}_+ \to \mathbf{R}_+$ with $f(0) = 0,\ f' > 0,\ f'' < 0,\ f'(0) = \infty,\ \lim_{z\to\infty} f'(z) = 0.$ The corresponding technology is

$$Y = \{(y_1,y_2,y_3) \in \mathbf{R}^3 \ \mid\ y_1 \leq 0,\ y_2 \leq 0,\ y_3 \leq f(-y_1)\}.$$

CONSUMERS: There are three types of consumers, types I, II and III.

(I) There are k consumers of type I. A consumer i of type I has preferences represented by the utility function $u_i(x_1,x_2,x_3) = x_3$ for consumption bundles $(x_1,x_2,x_3) \in \mathbf{R}^3_+$. Each such consumer has endowment $\omega_i = (0,0,0)$ and owns the share $\theta_i = 1/k$ of the firm.

(II) There are m consumers of type II. A consumer i of type II has preferences represented by the utility function $u_i(x_1,x_2,x_3) = x_3$ for consumption bundles $(x_1,x_2,x_3) \in \mathbf{R}^3_+$. Each such consumer has endowment $\omega_i = (0,1,0)$ and does not own shares of the firm.

(III) There are n consumers of type III. A consumer i of type III has preferences represented by the utility function $u_i(x_1,x_2,x_3) = x_1x_2$ for consumption bundles $(x_1,x_2,x_3) \in \mathbf{R}^3_+$. Each such consumer has endowment $\omega_i = (1,0,0)$ and does not own shares of the firm.

Consider a competitive equilibrium of Terra Nova's economy (p^*,x^*,y^*) (where the x's are quantities consumed and the y's are quantities of the input) with the price system $p^* = (1,p_2^*,p_3^*)$.

SHOW:

(a) For a consumer i of type III, $x_i^* = (1/2,\ x_{i2}^*,0)$ with $p_2^*x_{i2}^* = 1/2$.

(b) Market clearing for good 2 requires that for a consumer of type III either $kx_{i2}^* = m$ or $x_{i2}^* = m/k$.

(c) Using the findings in (a) and (b) above, determine p_2^* .

(d) Market clearing for good 1 yields the equilibrium input $z^* = k/2$.

(e) The first-order condition for the maximization of the profit $\pi(z) = p_3^*f(z) - z$ at $z^* = k/2$ yields $p_3^* = 1/f'(k/2)$.

(f) Use the results in (c) and (e) to determine the equilibrium consumption of a consumer of type II.

(g) Use the equilibrium profit $\pi(z^*) = p_3^*f(z^*) - z^*$ when $z^* = k/2$ and the value obtained for p_3^* to determine the equilibrium consumption of a consumer of type I.

(h) Determine the competitive equilibrium for the special case $f(z) = 2z^{1/2}$.

Problem 3 (60 minutes)

Consider the country of Tritonia and suppose that the government there can fix the level of inflation p by an appropriate choice of monetary policy. Similarly assume that the Council of Unions and Employers (CUE) of Tritonia can set the nominal wage increase w. (The CUE is assumed to be fully independent of the government and cannot be coerced into a particular choice by the government). We assume that $0 \leq p \leq 10$ as well as $0 \leq w \leq 10$. In Tritonia the growth rate of the real GDP, y, is directly related to the rate of inflation p and to the nominal wage increase w through the equation

$$y = 10 + (p - w).$$

The CUE would like *real wages* to remain constant, i.e. it would like the increase in the nominal wage rate to be equal to the rate of inflation, $w = p$. Therefore it is assumed that the CUE policy is driven by the payoff function

$$\pi_C(w,p) = -(w-p)^2.$$

For the government of Tritonia (representing its electorate) the growth over time of the GDP is more important than the rate of inflation. This is reflected in its payoff function which is given by

$$\pi_G(y,p) = y - p/2.$$

The CUE and the government of Tritonia interact as follows. First, the CUE selects w, the rate of increase in the nominal wage. Subsequently, after observing the choice of w by CUE the government determines its monetary policy, thereby setting the rate of inflation p.

(a) Formulate the macroeconomic policy setting problem as a game in extensive form.

(b) Determine all Subgame Perfect Nash Equilibria (SPNE) of this extensive form representation. In particular determine the SPNE payoff levels for both the government and CUE. What is the relationship between backward induction and rational expectations in this game? Explain in detail.

(c) Present this game in strategic, or normal, form. Does the game in this form have Nash Equilibria different from the SPNE you have determined in (b), and if it does then what are they?

(d) Suppose that the government could commit to a particular monetary policy before the CUE sets its increase in the nominal wage. What inflation rate p would it set? How would the payoffs of the government and of CUE compare to the SPNE payoff levels determined in (b)? Explain in detail.

Problem 4 (30 minutes)

A consumer with income Y consumes two goods, X_1 and X_2, which he can purchase at prices P_1 and P_2. His preferences can be represented by the utility function

$$U(X_1, X_2) = X_1^{\alpha} X_2^{1-\alpha}, \text{ where } 0 < \alpha < 1.$$

Suppose that between two consecutive periods the price of the first good declines from P_1^0 to P_1^1 while the price of the second good remains unchanged.

(a) Derive an expression for the change in the Marshallian consumer surplus. The expression you derive should depend only on the income, prices and the parameter α.

(b) Derive a similar expression for the compensating variation of the price change. (Notice the hint below).

(c) Derive a similar expression for the equivalent variation of the price change. (Notice the hint below).

(d) Discuss how the three welfare measures you have derived in (a) - (c) and the differences between them depend on α and on the size of the change in price.

(Hint: Recall that compensating variation and equivalent variation measure welfare changes along Hicksian demand curve).

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PART II

May 29, 2008

Please answer all 4 questions. Notice the time allotted to each question.

Problem 1 (60 minutes)

Consider the textbook multiple regression model

$$y = \mathbf{X}\beta + \varepsilon, \qquad \text{with } \varepsilon \sim IID(0, \sigma^2 I), \tag{1}$$

where $\mathbf{X}$ is an $n \times k$ matrix with full column rank. Assume that no intercept is contained in $\mathbf{X}$ and that the data have been demeaned.

(a) Is $\widehat{\beta}_{OLS}$ a consistent estimator of β in this setting? Please explain in detail how you arrived at your answer.

(b) Is $\widehat{\beta}_{OLS}$ an efficient estimator of β in this setting? Please explain in details how you arrived at your answer.

(c) Now suppose that you had access to an additional m observations on top of the previous n. However, each of the additional m observations of the k-vector of the covariates is now equal the non-zero k-vector $(x^*, x^*, \ldots, x^*)$. Given the fact that there is no intercept in the model, the data have to be demeaned anew prior to estimating β using the full sample of $n + m$ observations. Show that the $(l, j)^{\text{th}}$ element of $\mathbf{X}'\mathbf{X}$ for the newly demeaned dataset equals

$$\sum_{i=1}^{n} \mathbf{X}_{i,l}\mathbf{X}_{i,j} + \left(\frac{m}{m+n}\right)^2 n x^* x^* + \left(\frac{n}{m+n}\right)^2 m x^* x^* \tag{2}$$

Hint: $\overline{\mathbf{X}}_j$ now equals $\frac{m}{m+n} x^*$ when the additional m observations are added to the data set.

(d) Now, suppose that you conduct all of your asymptotic analysis on $m \to \infty$. Is $\widehat{\beta}_{OLS}$ consistent? What will happen to the variance of $\widehat{\beta}_{OLS}$ as m becomes large? Comment on the relationship between this result and your answer in the first two parts of this question.

(e) Explain what is happening in this situation. What does this suggest about asymptotic analysis in general?

Problem 2 (30 minutes)

(a) Discuss briefly how potential departures from assumptions [1] - [5] of the Normal/Linear regression model (see table 1 below) are likely to influence the unreliability of inference when testing the hypotheses

$$H_0 : \beta_1 = 0, \quad \text{vs.} \quad H_1 : \beta_1 \neq 0. \tag{1}$$

Hint: Explain how departures from such assumptions will affect the test for the hypotheses in (1), where

$$\widehat{\beta}_1 = \frac{\sum_{t=1}^{T}(x_t - \overline{x})(y_t - \overline{y})}{\sum_{t=1}^{T}(x_t - \overline{x})^2}$$

is the maximum likelihood estimator of β_1, where $\overline{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$ and $\overline{x} = \frac{1}{T}\sum_{t=1}^{T} x_t$.

(b) Explain how the presence of time trends in the data $\{(x_t, y_t),\ t = 1, 2, \ldots T\}$ will render the R^2 of the regression in table 1, completely unreliable.

(c) Explain how the presence of time trends in the data $\{(x_t, y_t),\ t = 1, 2, \ldots T\}$ will render the t-test for (1) , completely unreliable.

(d) How would one respecify the Normal/Linear regression in table 1 to take into account linear time trends in the data $\{(x_t, y_t),\ t = 1, 2, \ldots T\}$?

Table 1 - The Normal/Linear Regression (LR) Model	
Statistical GM:	$\boxed{y_t = \beta_0 + \beta_1 x_t + u_t,\ t \in \mathbf{T},}$
[1] Normality:	$(y_t \mid X_t = x_t) \backsim N(.,.)$,
[2] Linearity:	$E(y_t \mid X_t = x_t) = \beta_0 + \beta_1 x_t$, linear in x_t,
[3] Homoskedasticity:	$Var(y_t \mid X_t = x_t) = \sigma^2$, free of x_t,
[4] Independence:	$\{(y_t \mid X_t = x_t),\ t \in T\}$ is an independent process,
[5] t-variance:	$\theta := (\beta_0, \beta_1, \sigma^2)$ do not change with t.

Problem 3 (45 minutes)

Consider a Ramsey-type model with no technological change and zero population growth (so $L_t = L$, a constant). Output is

$$Y_t = AK_t^{\alpha}L^{1-\alpha} \,. \tag{1}$$

The budget constraint is

$$\dot{K}_t = \overline{q}_t[AK_t^{\alpha}L^{1-\alpha} - C_t] - \delta K_t \tag{2}$$

in which $\overline{q}_t \equiv q_t^{1-\alpha}$ and $q_t = e^{zt}$ .Think of $\overline{q}_t$ as indicating one unit of time t capital is as productive as $\overline{q}_t$ units of time 0 capital. This indicates that technological progress is embedded in new capital.

Let $\widetilde{k} \equiv K/Lq$. From this definition and equation (2) it follows that $\dot{\widetilde{k}}_t = A\widetilde{k}_t^{\alpha} - c_t e^{-\alpha zt} - \widetilde{\delta}\widetilde{k}_t$, where $c_t \equiv C_t/L_t$ is per capita consumption, and $\widetilde{\delta} \equiv z + \delta$.

(a) Instantaneous utility is $u(c_t) = [c_t^{1-\gamma}/(1-\gamma)]$. The command optimization problem is

$$\max_{c_t} \int_0^{\infty}[(c_t)^{1-\gamma}/(1-\gamma)]e^{-t\theta}dt,$$

Subject to $\dot{\widetilde{k}} = A\widetilde{k}_t^{\alpha} - c_t e^{-(\alpha z)t} - \widetilde{\delta}\widetilde{k}_t$.

Let $\widetilde{c}_t \equiv c_t e^{-\alpha zt}$. This optimization problem is equivalent to

$$\max_{\widetilde{c}_t} \int_0^{\infty}[(\widetilde{c}_t)^{1-\gamma}/(1-\gamma)]e^{-t\widetilde{\theta}}dt,$$

Subject to $\dot{\widetilde{k}} = A\widetilde{k}_t^{\alpha} - \widetilde{c}_t - \widetilde{\delta}\widetilde{k}_t$, $\widetilde{\theta} \equiv \theta - \alpha z(1-\gamma)$.

Solve this latter problem and determine steady state values for $\widetilde{k}$ and $\widetilde{c}$.

(b) Find the steady state growth rates of $k \equiv K/L$, $y \equiv Y/L$, and $c \equiv C/L$.

(c) What are the steady state growth rates of the real wage and the marginal product of capital?

Problem 4 (45 minutes)

Consider a two-period model in which each individual has at time t an endowment of a non-storable output. There are two types of individuals with n members of each type. The endowment output an individual of type i has at time j is e_{ij} . So the endowment profile is: Individual of type 1: $e_{11},\ e_{12}$; and individual of type 2: $e_{21},\ e_{22}$.

Utility for each type is given by $U(c_{i1}, c_{i2}) = \ln(c_{i1}) + \ln(c_{i2})/(1+\theta)$ where c_{ij} is the consumption of an individual of type i at time j .

There is a loans market at time 1 and the real interest rate is r .

The budget constraint at time 1 is: $c_{i1} + b_i^d = e_{i1}$, where b_i^d is the demand for bonds by an individual of type i at time 1. The budget constraint at time 2 is: $c_{i2} = b_i^d(1+r) + e_{i2}$, and the combined lifetime budget constraint is: $c_{i1} + c_{i2}/(1+r) = e_{i1} + e_{i2}/(1+r)$.

(a) Each individual seeks to maximize lifetime utility, given r and the lifetime budget constraint. Determine c_{i1} and the bond demand b_i^d as functions of r and the individual's endowments.

For the remainder of this problem assume that the endowments are:

For individual of type 1: $e_{11} > 0$ and $e_{12} = 0$
for individual of type 2: $e_{21} = 0$ and $e_{22} > 0$.

(b) Total bond demand is $n(b_1^d + b_2^d)$. In equilibrium this sum must be zero. Find the equilibrium interest rate.

(c) Suppose there is an increase in e_{22} , and this increase is known to all individuals before decisions are made in period 1. How does this change affect the equilibrium interest rate? Explain.

(d) Let $B_1^d = nb_1^d$ and let $B_1^s = -nb_2^d$. Graph bond demand and supply as functions of r . Explain your answer to (c) by showing and explaining the shift of the relevant curve.

(e) Suppose there is an increase in e_{11} . How will this increase affect the equilibrium interest rate? Explain.

(f) At time 1, $Y^d = c_{11}^d + c_{21}^d$ and $Y^s = e_{11}$. Graph the supply and demand as functions of r . Explain your answer to (e) by showing and explaining the shifts of relevant curve.