Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam

Fall 2009

PART II

November 5, 2009

Please answer all 4 questions. Notice the time allotted to each question.

1. (30 minutes)

Consider a model of entry. Firm 1 is the incumbent and it chooses its capital level k_1 first. Upon observing k_1 firm 2 decides whether to enter the market or not, and if it decides to enter it determines its level of capital k_2. Given k_1 and k_2 firm i's profits is given by $\pi_i(k_1, k_2) = k_i(1 - k_i - k_j), i = 1, 2 and i \neq j$.

(a) Suppose that firm 2 has no entry cost. Find the capital level of each firm in the subgame perfect equilibrium.

(b) Now suppose that the cost of entry is equal to 1/8. Find the level of capital of each firm in the subgame perfect equilibrium.

2. (60 minutes)

Part A.

Consider a two-person pure exchange economy with two consumers, $i = 1, 2$. The preferences of these consumers can be represented by the utility functions $u_1(x_1^1, x_1^2) = \min\{x_1^1, x_1^2\}$ and $u_2(x_2^1, x_2^2) = \min\{x_2^1, x_2^2\}$, and their initial endowments are $\omega_1 = (5, 1)$ and $\omega_2 = (3, 4)$.

Carefully draw an Edgeworth box diagram and in it draw precisely:

(a) The set of all Pareto allocations.

(b) Indicate the set of all competitive equilibria allocations and the price-line associated with each.

(c) The set of all allocations in the core.

Part B.

Answer (a), (b) and (c) above when the initial endowments are the same as above but the preferences are represented by the following utility functions: $u_1(x_1^1, x_1^2) = x_1^1 + x_1^2$ and $\mathrm{u}_2(x_2^1, x_2^2) = \begin{cases} 1 & if x_2^1 \leq x_2^2 \\ \\ 0 & if x_2^1 > x_2^2 \end{cases}$.

3. (45 minutes)

Consider a consumer with income m who consumes n goods, $x_1, x_2, ...x_n$, the prices of which are $p_1, p_2, ...p_n$, respectively. The preferences of this consumer can be represented by a utility function $u(x_1, x_2, ...x_n)$. In that context:

(a) Define the consumer's indirect utility function. Is it homogeneous, and if it is then of what degree?

(b) Define the expenditure function and prove that it is concave in the prices.

(c) Suppose the consumer consumes just two goods his indirect utility function is given by $\mathrm{v}(\mathrm{p}_1, p_2, m) = \frac{m}{p_1+p_2}$.

(c.1) What is the direct utility function in this case?

(c.2) What is the expenditure function?

(c.3) What is the Marshallian (regular) demand function for good 1?

(c.4) What is the Hicksian (compensated) demand function for good 2?

(c.5) What is the Slutsky equation?

4. (45 minutes)

Three girls, Gloria, Diva and Trivia, are students at Tech who share an apartment in town. At night they can either study in their respective rooms or they can go to the corner bar to have a drink. The utility each gets from studying is 0. The cost to each of going to the bar is 1. The utility of each from drinking at the bar is 2 if not all three of them are there at the same time, and is 1/2 if all three are present.

In answering the following questions try to be as complete as possible in your analysis.

(a) Formulate the situation described above as a three person game in normal (strategic) form. State precisely the pure strategies and the payoff function of each player. Derive ALL pure strategy Nash equilibria of the game.

(b) For this problem we say that a player plays a mixed strategy if the probability at which she plays at least one pure strategy is strictly between 0 and 1.

Show that for this game there is a unique Nash equilibrium in mixed strategies and in it each girl goes to the bar with probability $p = \frac{1}{3}\sqrt{6}$.