Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam

Spring 2009

PART I

June 1, 2009

Please answer all 4 questions. Notice the time allotted to each question.

Problem 1. (60 minutes)

Instructions:

Consider the textbook multiple regression model

$$y = X\beta + \varepsilon, \text{ with } \varepsilon \sim IID(0, \sigma^2 I), \tag{(1)}$$

where X is an n x k matrix with full column rank.

1. Show that the inclusion of a set of j **irrelevant** variables, Z, into the regression in (1) leads to an OLS estimator of β that is inefficient relative to an estimator of the model where Z was not included when $X'Z \neq 0$.

2. Show that the exclusion of a set of j **relevant** variables, Z, from the regression in (1) leads to an OLS estimator of β that is inconsistent when $X'Z \neq 0$. Of what quantity **does this** OLS estimator provide a consistent estimate for?

3. Show that when $X'Z = 0$ the inclusion of Z into the regression in (1) has no adverse impact on the consistency or variance of the OLS estimate of β.

4. Derive the conditional variance of the OLS estimator if one had instead assumed $\varepsilon \sim ID(0, \Omega)$ where Ω is a diagonal matrix.

5. Is the OLS estimator $BLUE$ in the setting of question 4? Support your answer with a formal mathematical argument.

6. Prove that the inclusion of $Z = X - \Omega^{-1}XC'$, where $C = X'\Omega^{-1}X(X'\Omega^{-2}X)^{-1}$, into the regression

$$y = X\beta + \varepsilon, \text{ with } \varepsilon \sim ID(0, \Omega), \tag{(2)}$$

where Ω is diagonal, **when using** OLS provides an estimate with the same efficiency as that of the GLS estimate. **Hint:** let $B = \Omega^{-1}XC'$ and rewrite your regression equation as $y = (X - Z)\beta + Z(\gamma + \beta) + \varepsilon$.

7. What does this result imply about overspecifying a regression model when the error terms are heteroscedastic?

8. Let $\widehat{\beta}_{FGLS}$ denote the feasible GLS estimator of the model in (2), using a consistent estimator for Ω, which you can denote $\widehat{\Omega}$. Additionally, let $\widehat{\beta}_{FZ}$ be the feasible OLS estimator for the model suggested in (2) where the new regressor included is $\widehat{Z} = X - \widehat{\Omega}^{-1}X\widehat{C}'$ and $\widehat{C}$ is defined as having Ω replaced with $\widehat{\Omega}$. Prove that $\widehat{\beta}_{FGLS} = \widehat{\beta}_{FZ}$.

9. What does this result suggest?

Problem 2 (30 minutes)

In the course of examining the statistical adequacy of a regression model, an analyst evaluates the p-value for a hypothesis test on the coefficient(s) of an auxiliary regression.

1) In what fundamental way does this test differ from a hypothesis test on a coefficient, or combination of coefficients, in the original regression model?

2) What kind of auxiliary regression would be suitable for examining the adequacy of the assumption with regard to model serial dependence – or "model error nonautocorrelation" in the usual regression formulation?

3) If the auxiliary regression of part 2 above indicates the existence of a serious problem with this assumption, describe two approaches for ameliorating or eliminating the problem.

4) Discuss the extent to which elaborating the estimated model, as a result of either of the approaches you discuss in part 3 above, constitutes "data mining" and invalidates the statistical inferences obtained.

Problem 3. (45 minutes)

Consider the following problem: There is no population growth and each individual in the economy seeks to maximize lifetime utility. The time 0 problem is

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t), \text{ such that } k_{t+1} = Ak_t{}^{\alpha} - c_t, k_0 \text{ is given and } 0 < \alpha < 1. \quad ([1])$$

Work effort is fixed at $l = 1$, so the capital stock and the capital stock per unit of labor are equal (as are output and output per unit of labor). Output at time t is $y_t = Ak_t^{\alpha} l_t^{1-\alpha} = Ak_t^{\alpha}$. There is a loans market, but in equilibrium the interest rate is such that the only saving is in the form of capital (time t saving is k_{t+1}). For this reason one can simplify the problem by ignoring bonds. $\beta = 1/(1 + \theta)$.

Let $u(c) = \ln(c)$. Then the utility maximization problem is equivalent to

$$\max_{c_0} \{ \ln(c_0) + \beta V(k_1) \text{ , such that } k_1 = Ak_0^{\alpha} - c_0 \}, \quad ([2])$$

in which $V(k_t) \equiv E + [\alpha \ln(k_t)]/(1 - \alpha\beta)$, and E is a constant.

(a) Solve the problem in equation [2], showing that optimal consumption (c_0) and saving (k_1) are constant fractions of $y_0 = Ak_0^{\alpha}$.

(b) The equations in (a) hold for each time t. Use this fact to determine steady state levels of capital, output, and consumption. Label these as k^*, y^*, and c^*. Use a graph in $(k_{t+1}, k_t)-$space to show how k^* is determined.

(c) The interest rate at which the demand for bonds is zero is $r_t = MPK_t - 1$. Find the steady state MPK (marginal product of capital) and the steady state value for r. Explain the economic intuition for this result.

(d) Find the steady state real wage (marginal product of labor). (Note $y = Ak^{\alpha} l^{1-\alpha}$ and $l = 1$).

(e) Suppose initially that $k_0 < k^*$, describe the time path of the marginal product of capital and labor as the economy converges to the steady state. (Show this graphically.)

Now let's introduce government purchases into the model. Suppose government purchases are a constant fraction of income, $g_t = \tau y_t = \tau Ak_t^{\alpha}$ and are financed by taxing income at the rate of τ, so after-tax income is $(1 - \tau)Ak_t^{\alpha}$.

(f) For a non-zero tax rate, what are the steady state values for k, y and c?

(g) Assume that initially the economy is in the steady state at a non-zero level of government purchases. Suppose there is an increase in government purchases and hence in the tax rate. Show graphically the effect this will have on steady state level of k. Describe the time path of the MPK and the real wage (the MPL) as the economy converges to its new steady state.

Problem 4 (45 minutes)

We have the following system:

Aggregate Demand: $y_t = (m_t - p_t) + u_t$ Aggregate Supply: $y_t = -({}_{t-1}w_t - p_t)$

$$\text{Wage Setting Rule: } {}_{t-1}w_t = E_{t-1}p_t.$$

All variables are in log. y_t is output, m_t is money supply, p_t is price level, u_t is an exogenous demand shock, ${}_{t-1}w_t$ is nominal wage set at the end of period $t-1$ for period t. The exogenous demand shock u_t is a white noise.

(a) Briefly explain the three equations in words.

(b) Solve for the equilibrium wage ${}_{t-1}w_t$ and price p_t in terms of the exogenous variables m_t and u_t. Briefly explain your result. .

(c) Based on (b), solve for the equilibrium output y_t. Can a money supply rule of the form $m_t =_{i=1}^{\infty} a_i u_{t-i}$, which is based on the demand shocks observed up to period $t-1$, stabilize output? If your answer is yes, solve for the rule that minimizes the variance of output.

(d) Now change the second and third equations to:

$$\text{Aggregate supply: } y_t = -\tfrac{1}{2}({}_{t-1}w_t - p_t) - \tfrac{1}{2}({}_{t-2}w_t - p_t)$$

$$\text{Wage setting rule: } {}_{t-1}w_t = E_{t-1}p_t,\ {}_{t-2}w_t = E_{t-2}p_t$$

where ${}_{t-1}w_t$ is the nominal wage set at the end of period $t-1$ for the period t, and ${}_{t-2}w_t$ is the nominal wage set at the end of period $t-2$ for the period t. Explain the two new equations in words. How is the wage contract different from the original one?

(e) Solve for the equilibrium wage ${}_{t-1}w_t$ and price p_t in terms of the exogenous variables m_t and u_t.

(f) Based on (e), solve for the equilibrium output y_t. Can a money supply rule of the form $m_t =_{i=1}^{\infty} a_i u_{t-i}$, which is based on the demand shocks observed up to period $t-1$, stabilize output? If your answer is yes, solve for the rule that minimizes the variance of output.

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PART II

June 4, 2009

Please answer all 4 questions. Notice the time allotted to each question.

Problem 1. (30 minutes)

Consider a market that currently has two firms, 1 and 2. Firm 3 is a potential entrant into this market. All firms produce an identical product. Firms 1 and 2 simultaneously decide how much to produce, q_1 and q_2. After observing these quantities firm 3 decides whether to enter the market or not and how much to produce, q_3, if it does enter. Since the production process takes some time all quantities produced by all three firms get to the market at the same time. Let $c > 0$ be the constant marginal cost of each firm and let $p(q) = a - (q_1 + q_2 + q_3)$, $a > c$, be the market inverse demand curve. The cost of entry for firm 3 is given by $F \geq 0$.

(a) If $F > a^2$ (i.e. F is large enough so that firm 3 will not enter), what are q_1 and q_2 in equilibrium?

(b) If $F = 0$, what are q_1, q_2 and q_3 in a subgame perfect Nash equilibrium?

Problem 2. (75 minutes)

PART 1

Consider a pure exchange economy with $L \geq 2$ goods labeled $k = 1, \ldots, L$ and $n \geq 2$ consumers labeled $i = 1, \ldots, n$. All consumers have the same endowment bundle $\omega_0 = (\omega_0^1, \ldots, \omega_0^L) \gg 0$. That is, for each consumer i, the initial endowment is $\omega_i = \omega_0$. All consumers have Cobb-Douglas preferences, which may differ across consumers: There exist numbers $\alpha_i^k > 0$ for $i = 1, \ldots, n$; $k = 1, \ldots, L$ such that consumer i has utility representation

$$u_i(x_i^1, \ldots, x_i^L) = (x_i^1)^{\alpha_i^1} \cdot (x_i^2)^{\alpha_i^2} \cdots (x_i^L)^{\alpha_i^L}$$

or alternatively

$$\ln u_i(x_i^1, \ldots, x_i^L) = \sum_{k=1}^{L} \alpha_i^k \ln x_i^k$$

for $x_i = (x_1^1, \ldots, x_i^L) \in \mathbb{R}_+^L$.

We further assume that the α_i^k are normalized so that $\sum_{k=1}^{L} \alpha_i^k = 1$ for each consumer i.

Finally, set $\alpha^k = \dfrac{1}{n} \sum_{i=1}^{n} \alpha_i^k$ for $k = 1, \ldots L$. That is, $\alpha^1, \ldots, \alpha^L$ constitute the utility parameters for an "average consumer".

(a) Show that in equilibrium, $x_i^k = \dfrac{\alpha_i^k}{\alpha^k} \cdot \omega_0^k$ for $i = 1, \ldots, n$; $k = 1, \ldots, L$. Hint. At any price system, all consumers have the same income. Write down and solve the market clearing conditions for all goods.

(b) Suppose j happens to be an average consumer, that is, $\alpha_j^k = \alpha^k$ for $k = 1, \ldots, L$. Does j trade in equilibrium?

PART 2

Next consider two countries, North Equitania and South Equitania, without trade between the two countries. Each country's inhabitants are consumers like above.

Specifically, North Equitania's economy consists of two types, A and B, of consumers. A consumer i of type A has endowment ω_0 and utility parameters $\alpha_i^k = \dfrac{2k}{L(L+1)}$. A consumer i of type B has endowment ω_0 and utility parameters $\alpha_i^k = \dfrac{1}{L}$.

Specifically South Equitania's economy consists of two types, B and C, of consumers. A consumer i of type C has endowment ω_0 and utility parameters $\alpha_i^k = \dfrac{2(L+1-k)}{L(L+1)}$.

There are $n_A \geq 1$ consumers of type A and $n_B \geq 1$ consumers of type B in North Equitania. There are $n_C = n_A$ consumers of type C and n_B consumers of type B in South Equitania.

Let x_i^N denote the competitive equilibrium consumption of an inhabitant i of North Equitania when each country has its own separate economy. Let x_i^S denote the competitive equilibrium consumption of an inhabitant i of South Equitania when each country has its own separate economy.

Suppose North and South Equitania consider integrating their economies, that is forming a pure exchange economy that includes the inhabitants of both countries. Let x_i^I denote the competitive equilibrium consumption of consumer i in the integrated economy.

(c) Which are the "average consumers" of the integrated economy?

(d) Would the "average consumers" of the integrated economy gain ($u_i(x_i^I) > u_i(x_i^N)$ if i resides in North Equitania; $u_i(x_i^I) > u_i(x_i^S)$ if i resides in South Equitania) or lose ($u_i(x_i^I) < u_i(x_i^N)$ if i resides in North Equitania; $u_i(x_i^I) < u_i(x_i^S)$ if i resides in South Equitania) from economic integration of the two countries? Explain!
Hint. Recall (b).

(e) It cannot be the case that all consumers lose from economic integration. Why?

Problem 3 (30 minutes)

A community consists of two households ($h = 1, 2$) located on an (otherwise unused) lake. Each household generates one unit of waste, any part of which may be dumped into the lake at no resource cost. Denote this fraction of the waste by s_h, where $0 \leq s_h \leq 1$ (s represents sewage).

The part of the waste not dumped into the lake, $1 - s_h$, must be disposed of by an alternate technology on land at a resource cost given by:

$$c(s_h) = (1 - s_h)^2.$$

Each household h has the same utility function, given by:

$$U^h(s_1, s_2) = 10 - c(s_h) - (s_1 + s_2).$$

(a) What value will household h choose for s_h and what level of utility will it attain in each of the following two cases:

(a.1) Household h assumes that the other household's decision will be independent of s_h.

(a.2) Household h assumes that the other household's decision will depend on s_h.

(b) If a social planner were to choose the level of sewage for each household so as to maximize total utility then what levels will he choose and what utility will each household attain?

(c) Provide an economic explanation for the difference in the results in (a) and in (b).

Problem 4. (45 minutes)

Consider the sale of tickets to football games at a certain state university. The demand for tickets can be separated into two groups. The demand curve for students is $Q_S = 36 - 2P_S$ and the demand curve for non-students is $Q_{NS} = 88 - P_{NS}$, where quantities are measured in thousands. The capacity of the stadium is 60 (thousand) seats.

(a) At present the university can exercise price discrimination. What price should it charge each of the two groups if it wishes to maximize its revenue? How many tickets will be sold to each group? What will be the total consumer surplus?

(b) Suppose now that lawmakers in the state capital were successful in passing a law that prevents the university from charging different prices to different people.

(b.1) What is the aggregate demand facing the university?

(b.2) What (uniform) price should the university charge in this case if, as before, it wishes to maximize total revenue? How many tickets will it sell? What will be the consumer surplus in this case?

(c) Some lawmakers claim that the price derived in (b.2) above is "too high" and propose that the university will have to sell all tickets at a price $P = 8$. However, since at that price total quantity demanded is larger than the capacity of the stadium, the university will have to allocate tickets to the two groups of customers.

(c.1) Assume that tickets can not be resold. Calculate the minimum and maximum total consumer surplus from the possible allocations of tickets.

(c.2) Now suppose that tickets can be resold without transaction cost. What will the size of total consumer surplus be in this case? What will be the size of consumer surplus of students?