Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam

January 2012

PART I

January 9, 2012

Please answer all 4 questions. Notice the time allotted to each question.

Problem 1. (45 minutes)

A consumer consumes two goods, 1 and 2. His preferences over consumption bundles $(x_1, x_2) \in \mathbf{R}^2_+$ can be represented by the utility function

$$u(x_1, x_2) = \min\{2x_1 + x_2,\ x_1 + 2x_2\}.$$

Let p_i be the price of good $i,\ i = 1, 2$ and let m be this consumer's income.

Part A:

(a) Derive this consumer's Marshallian demand for each good (hint: be careful).

(b) Derive his indirect utility function and his expenditure function.

(c) Derive the Hicksian (compensated) demand for each good.

Part B:

Answer questions (a) (b) and (c) above in case the preferences of this consumer can be represented by the utility function

$$u(x_1, x_2) = \max\{2x_1 + x_2,\ x_1 + 2x_2\}$$

Problem 2. (45 minutes)

Consider a pure exchange economy with two consumers labeled $i = 1, 2$ and two goods, x and y. A consumption bundle of consumer i assumes the form $(x_i, y_i) \in \mathbf{R}^2_+$. The preferences of consumer 1 can be represented by the utility function

$$u_1(x_1, y_1) = \begin{cases} \min\{x_1, y_1\} \text{ if } x_1 \leq 8 \text{ or } y_1 \leq 8 \\ \\ 8 \text{ otherwise} \end{cases}$$

and those of consumer 2 can be represented by the utility function

$$u_2(x_2, y_2) = x_2 + y_2.$$

The initial endowments of the consumers are $\omega_1 = (20, 5)$ and $\omega_2 = (0, 5)$.

(a) Draw the Edgeworth box of this exchange economy.

You can answer questions (b) through (d) either diagrammatically or algebraically but in either case BE PRECISE!

(b) What is the set of Pareto efficient allocations of this economy?

(c) What are all competitive equilibria (both prices and allocations) of this economy.

(d) What are the core allocations of this economy?

(e) State the First Theorem of Welfare Economics and determine whether it holds true in this economy.

Problem 3. (60 minutes)

Examine the following model of infinitely-repeated Bertrand interaction between two firms. In each period there is a probability $\lambda \in (0,1)$ of a "high-demand" state in which the demand is $x(p)$ and a probability $(1-\lambda)$ of a "low-demand" state in which the demand is $\alpha x(p)$, where $\alpha \in (0,1)$ and $x(p)$ is a continuous and decreasing function of the price p. In each period when a firm makes a decision about the price it will charge at that period it already knows the true state of the demand in that period. For each of the two firms the unit cost of production is $c > 0$ and the periodic discount factor is $\delta \in (0,1)$. In addition, assume that both firms observe the outcome of every past period.

Consider Nash reversion strategies of the following form: Charge the price p_H in a high-demand state if there has been no past deviation, charge the price p_L in a low-demand state if there has been no past deviation, and set the price in either state equal to c if a deviation has occurred previously.

(a) Prove that if δ is sufficiently high, then there is a subgame perfect Nash equilibrium (SPNE) in which each firm sets $p_H = p_L = p^m$, where p^m is the unique monopoly price in both states.

(b) Show that for some $\underline{\delta} > \frac{1}{2}$ a firm would want to deviate from p^m in a high demand state whenever $\delta < \underline{\delta}$.

(c) for $\delta \in [\frac{1}{2}, \underline{\delta})$, identify the highest price p_H that the firms can sustain in a SPNE and check that they can still sustain the price $p_L = p^m$ in the low demand state.

(d) Show that if $\delta < \frac{1}{2}$ then both firms will set $p_H = p_L = c$ in a SPNE.

Problem 4. (30 minutes)

Consider the following static entry game. Firms 1 and 2 simultaneously decide whether or not to enter a new market. Denote by $a_i,\ i=1,2,$ firm i's decision, where $a_i=0$ means it does not enter and $a_i=1$ means it does enter. The firms' payoffs are given by

$$\pi_i(a_i,a_j)=a_i\cdot(S_i-a_j\cdot\triangle),\ i=1,2,\ i\neq j,$$

where $S_1,\ S_2$ and $\triangle$ are parameters. Derive the set of all Nash equilibria in both pure and mixed strategies in each of the following cases.

(a) $0<S_i<\triangle$ for both $i=1,2$.

(b) $0<S_i<S_j=\triangle$.

(c) $0<S_i=S_j=\triangle$.

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PART II

January 12, 2012

Please answer all 5 questions. Notice the time allotted to each question.

Problem 1. (30 minutes)

"In estimation, the method of Maximum Likelihood (ML) is based on the idea that the most representative estimate is the value of the parameter(s) that have the highest probability (likelihood) of giving rise to the observed data."

1. Explain this statement by relating the likelihood function to the distribution of the sample, in general terms.
2. Now consider the specific case of the *simple Bernoulli model*:

$$X_t \sim \text{BerIID}\left(\theta, \theta\left(1-\theta\right)\right), \quad t = 1, 2, \ldots, n. \tag{1}$$

   (a) Show the distribution of the sample and the likelihood function.

   (b) Derive the Maximum Likelihood Estimator $\widehat{\theta}_{MLE}$, and show that it is indeed a maximum.

   (c) State its finite sampling distribution.

   (d) Derive its expectation and variance.

   (e) Derive the *Fisher information* and show how it relates to $Var\left(\widehat{\theta}_{MLE}\right)$ and the *Cramer-Rao lower bound*.

   (f) In light of these results, state the *finite sample properties* of $\widehat{\theta}_{MLE}$ (in words and using formal mathematical notation).

   (g) State its asymptotic properties (in words and using formal mathematical notation).

Problem 2. (30 minutes)

Suppose that

$$
\begin{aligned}
& Y_i = \beta X_i + U_i \\
& U_i \sim IID \quad n\left(0, \sigma^2\right) \\
& X_i \neq 0, i = 1 \ldots N, \quad X_i \neq X_j, i \neq j
\end{aligned}
\tag{2}
$$

where n denotes the normal density and Y_i, X_i, and U_i are scalars. You can assume that $X_i, i = 1 \ldots N$ is fixed in repeated samples, and that $\underset{N\to\infty}{lim} \sum_{i=1}^{N} X_i^{-2}$ is finite.

Consider the estimator

$$
\tilde{\beta} \equiv \frac{1}{N} \sum_{i=1}^{N} \frac{Y_i}{X_i} \tag{3}
$$

1. Obtain the sampling distribution of $\tilde{\beta}$, i.e. its mean, variance, and the form of its distribution.
2. Examine formally if $\tilde{\beta}$ is a consistent estimator for β.
3. Show the explicit form of the OLS estimator (call it $\hat{\beta}$), and its variance.
4. Make a statement about the comparative magnitude of the two variances. You can try brute force (good luck...) or invoke a well-known theorem.
5. Is $\tilde{\beta}$ asymptotically efficient? Explain.
6. Make a statement about the comparative length of a $100\,(1-\alpha)\%$ confidence interval for β based on $\tilde{\beta}$ versus $\hat{\beta}$. Explain.
7. Given the value of σ^2, how would you test $H_0 : \beta = 3$ against $H_A : \beta \neq 3$?

Problem 3. (30 minutes)

Consider the standard unobserved effects model with only two time periods:

$$y_{it} = x_{it}\beta + \mu_i + \nu_{it}, \quad t = 1, 2, \quad i = 1 \ldots N \tag{4}$$

1. Show that in this case the Fixed Effects (FE) and First Difference (FD) estimates are numerically identical.

2. Despite their numerical equivalence, under what assumptions on ν_{it} would you prefer the FE estimator? Please provide a detailed mathematical derivation to back up your statement, and discuss it.

3. Under what assumptions on ν_{it} would you prefer the FD estimator? Please provide a detailed mathematical derivation to back up your statement, and discuss it.

Problem 4. (60 minutes)

A household maximizes a quadratic lifetime utility given by

$$U_t = E_t\left\{\sum_{i=0}^{\infty} \beta^i (C_{t+i} - C_{t+i}^2/2)\right\}.$$

The household can invest in an asset that pays a constant rate of return

$$A_{t+1} = (1+r)A_t + Y_t - C_t.$$

Initial wealth A_0 is given. For simplicity assume $\beta = (1+r)^{-1}$. The household faces income uncertainty.

(a) Determine time t consumption as a function of time t income and the expected time path of future income.

Now assume income follows an AR(1) process

$$Y_t = \rho Y_{t-1} + u_i$$

where $0 < \rho < 1$ and u_i is i.i.d., $N(0, \sigma^2)$. At time t the household knows $Y_{t-i},\ i = 0, 1, \ldots, t$ but has to predict $Y_{t+i}, i = 1, 2, \ldots$ Under these assumptions answer:

(b) Is the "surprise" to consumption, $C_{t+1} - E(C_{t+1})$ more volatile than the "surprise" to income, $Y_{t+1} - E(Y_{t+1})$? That is, does the Deaton's paradox apply here? Show your steps.

(c) What happens when $\rho = 1$, that is, when income level is a random walk? Explain your result intuitively.

(d) Does the household have any precautionary saving? Explain your result intuitively.

For the next question add the following assumptions:

(1) A tax is imposed on consumption with γ being the tax rate.

(2) At time t each household receives a real grant from the government equal to R_t.

(3) By way of eliminating the income effects of the tax, assume that $R_t = \gamma C_t$ (although each individual takes R_t as given exogenously).

Under these additional assumptions the new budget constraint is

$$A_{t+1} = (1+r)A_t + Y_t - (1+\gamma)C_t + R_t.$$

Now answer

(c) Will the imposition of a tax on consumption increase savings? Explain.

Problem 5. (30 minutes)

This problem consists of three statements. Determine if each is true or false and **justify your answers.** If a statement can be true or false, depending upon circumstances, explain why this is so. To answer these questions assume the real output and the real interest rate are constant. Consider an equilibrium in which long-run inflation is also a constant and write a model determining the current price level, P, and the nominal interest rate, R, (or equivalently a model of P and long-run inflation). The three statements are:

(a) A temporary tax cut financed with an increase in government bonds is inflationary.

(b) A temporary tax cut financed with additional money is inflationary.

(c) An open market purchase of government bonds by the monetary authority is equivalent to a helicopter drop of money.