Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam March 2013

PART I

March 29, 2013

Please answer all 4 questions. Notice the time allotted to each question.

<u>Problem 1</u> (60 minutes)

There is a private ownership economy $\mathcal{E} = \{(X^i, \succsim^i)_{i=1}^2, (Y^1), (\omega^i, \theta^i)_{i=1}^2\}$ with two consumers $i \in \{1, 2\}$, one firm, and two commodities $l \in \{1, 2\}$. Each consumer i chooses among commodity bundles in the set $X^i = \mathbb{R}^2_+$ according to his/her preferences $\succsim^i$ described by the Leontief utility function:

$$u(x_1^i, x_2^i) = \min\{x_1^i, x_2^i\}.$$

The initial endowment of consumer 1 is given by $\omega^1 = (4, 1)$ and that of 2 by $\omega^2 = (2, 2)$. Consumer i's share of the firm 1 is denoted by $\theta_1^i \in [0, 1]$ and $\theta_1^1 + \theta_1^2 = 1$. Suppose that the firm can produce commodity 2 by using commodity 1 according to the following technology:

$$Y^1 = \{(-y_1, y_2) \in \mathbb{R}^2 : y_2 \leq \frac{1}{2} \cdot y_1, 0 \leq y_1\}.$$

Let $p \in \mathbb{R}^2_+$, $p \neq 0$, be a price vector.

(a) Sketch the technology in a diagram and determine the firm's supply correspondence $y^1(p)$ as well as the profit function $\pi^1(p)$.

(b) Determine consumer i's Walrasian demand correspondence $x^i(p, \omega^i, \theta^i)$. Is it homogeneous of degree zero?

(c) Determine the Walrasian equilibriumm for the economy $\mathcal{E}$.
Use $p = (p_1, 1)$.

(d) Characterize the set of all Pareto-optimal allocations in the economy $\mathcal{E}$.

(e) Suppose now that the only feasible production plans are those of free-disposal, i.e. $Y^1 = -\mathbb{R}^2_+$. What is (are) the Walrasian equilibrium(a) in this situation?

Problem 2 (40 minutes)

A consumer's preferences are characterized by the expenditure function $e(p, u) = (p_1^r + p_2^r)^{1/r} u$.

(a) Are these preferences homothetic? Explain!

(b) Find the consumer's Marshallian demand function.

(c) Find the consumer's Hicksian demand function.

(d) Compute the Slutsky equation for good 1.

(e) Find the consumer's indirect utility function.

(f) How would you classify the goods in terms of income (e.g., inferior, normal) and does this depend on the value of r? Explain!

Problem 3 (30 minutes)

Consider a Cournot duopoly with firms $i = 1, 2$ facing a market inverse demand function $P(Q) = e^{-Q}$ for aggregate output $Q \geq 0$. Firm 1 chooses output $q_1 \geq 0$ and incurs costs $C_1(q_1) = 0$. Firm 2 chooses output $q_2 \geq 0$ and incurs costs $C_2(q_2) = q_2$. Aggregate output (industry output) is $Q = q_1 + q_2$.

(a) Determine the reaction function (best-response function) of firm 1.

(b) Determine the reaction function (best-response function) of firm 2.

(c) Determine the Nash equilibrium (Cournot-Nash equilibrium) pair (q_1^*, q_2^*).

(d) Does (q_1^*, q_2^*) maximize joint profit? Explain!

<u>Problem 4</u> (50 minutes)

Consider a price-taking firm i whose only input is labor. The firm's production function is

$$f(L) = \ln(1 + L)$$

where $L \geq 0$ is the firm's labor input. Let $P > 0$ denote the output price and $W > 0$ denote the input price (wage rate).

(a) Given P and W, formulate the firm's profit as a function $\Pi(L)$ of its labor input choice, L.

(b) Given P and L, determine

(i) the firm's maximal profit [by maximizing Π with respect to L];

(ii) the firm's profit maximizing labor input, as a function $L_i^D(P, W)$ of P and W;

(iii) the firm's profit maximizing output as a function $S_i(P, W)$ of P and W.

(c) Suppose the industry consists of 200 such firms, $i = 1, \ldots, 200$. Moreover, market demand is given by a linear demand function,

$$D(P) = 350 \cdot \ln 15 - P \text{ for } 0 \leq P \leq 350 \cdot \ln 15.$$

Finally, there is a fixed supply of labor, $\overline{L} = 2800$.

(i) Determine aggregate labor demand,

$$L^D(P, W) = \sum_{i=1}^{200} L_i^D(P, W).$$

(ii) Determine industry supply

$$S(P, W) = \sum_{i=1}^{200} S_i(P, W).$$

(iii) Find an output price P^* and a wage rate W^* such that both the labor and the output market are cleared, i.e.,

$$\begin{aligned} L^D(P^*, W^*) &= \overline{L}; \\ S(P^*, W^*) &= D(P^*). \end{aligned}$$

HINT. You can leave the expression $\ln 15$ as such.

Department of Economics and Department of Agricultural and Applied Economics

Macroeconomics Ph.D. Qualifying Exam

March 25, 2013, 9-11AM

Macroeconomics Question 1 **45 minutes**

A Simple Ramsey Model

There is no population growth and lifetime utility is

$$U = \int_0^\infty \ln(C_t) e^{-t\theta} \partial t \, , \; \theta > 0,$$

θ being the instantaneous rate of time preference. Output is given by the function

$$Y_t = AK_t^{1/2} ,$$

A being a constant productivity level.

Capital is accumulated through saving:

$$\frac{\partial K_t}{\partial t} = Y_t - C_t \, .$$

(a) Write down the optimality conditions (including the transversality condition) and determine the steady state values for consumption and capital.

(b) Linearize the model about the steady state and show that there is one stable root and one unstable root.

(c) Draw the phase diagram. Assume that initially K is below its steady state value. Show the time path of the economy as it converges to the steady state.

(d) Suppose the economy is initially in its steady state. Describe how an increase in A affects the steady state. In the phase diagram show the economy's path to the new steady state. Explain.

(e) Suppose the economy is initially in its steady state. Describe how an increase in θ affects the steady state. In the phase diagram show the economy's path to the new steady state. Explain.

Macro Question 2 **45 minutes**

Consider an endowment economy in which agents live for three periods and there are overlapping generations. At any point in time there will be (1) the young, (2) the middle-aged, and (3) the old. There is no population growth and therefore each generation is of equal size. Each member of generation *t* has a lifetime endowment vector of $(y_t^y, y_t^m, y_t^o) = (y, y, 0)$, superscripts indicating the stage in life. Output is perishable, so goods cannot be stored for future use. A generation t young person's lifetime utility is

$$U(c_t^y, c_t^m, c_t^o) = \ln c_t^y + [\ln c_t^m]/(1+\theta) + [\ln c_t^o]/(1+\theta)^2$$

(a) The time *i* real interest rate is r_i. A young person's utility optimization problem is to

$$\text{Max } U(c_t^y, c_t^m, c_t^o), \text{ s.t. } c^y + c^m/(1+r_t) + c^o/(1+r_t)(1+r_{t+1}) = y + y/(1+r_t)$$

Find the optimal values for (c_t^y, c_t^m, c_t^o) as function of r_i and *y*.

(b) There may be a stationary equilibrium for this model having the following characteristics. First, the interest rate is constant and second, each generation has the same lifetime consumption vector, so that $(c_t^y, c_t^m, c_t^o) = (c^y, c^m, c^o)$, a constant vector independent of time. In equilibrium, at each time *t* total consumption across the three generations must equal time *t* output. Find the stationary equilibrium interest rate and the lifetime consumption vector. (Hint: The interest rate may surprise you.)

(c) In the stationary equilibrium describe the path of saving across the lifespan of any given individual. Describe the economic activity of a middle aged person at any time t.

Now assume there is population growth. Let N_t be the population size of generation t. Assume $N_{t+1} = N_t(1+n)$.

(d) Consider a stationary equilibrium in which the interest rate is constant at *r*. What is the equilibrium interest rate in such a stationary equilibrium?

Problem 1 (30 minutes)

A. (a) Define and explain the notion of the likelihood function as it relates to the distribution of the sample.

(b) State the optimal properties of Maximum Likelihood Estimators (MLE) (finite sample and asymptotic), under the well-known regularity conditions.

B. (a) State and explain the Gauss-Markov theorem as it relates to the OLS estimators of (α_0, α_1) of the Linear model:

$$y_t = \alpha_0 + \alpha_1 x_t + \varepsilon_t, \; t = 1, 2, ..., n,$$

under the assumptions:

[1] $E(\varepsilon_t) = 0$, [2] $Var(\varepsilon_t) = \sigma^2$, [3] $E(\varepsilon_t \varepsilon_s) = 0, \; t \neq s, \; t = 1, ..., n,$

[4] $\sum_{t=1}^{n} (x_t - \overline{x})^2 \neq 0.$

C. (a) Explain why the Gauss-Markov 'optimal' properties of the OLS estimator of α_1 do not go far enough to enable one to construct a test for the hypotheses:

$$H_0 : \alpha_1 = 0 \text{ vs. } H_1 : \alpha_1 \neq 0.$$

(b) What additional assumption(s) would one need to construct such a test? Explain why.

Question 2 (30 minutes)

For a model with no intercept and one explanatory variable, at the observation level:

$$y_i = \beta x_i + \varepsilon_i, \ \ i = 1,2, \dots, N$$

a) Show that using the moment condition $\frac{1}{N}\sum_i x_i \varepsilon_i = 0$ results in the OLS estimator.

b) Show that using the moment condition $\frac{1}{N}\sum_i z_i \varepsilon_i = 0$ results in the IV estimator (where *z* is a valid and strong instrument).

c) Please illustrate what you would do to produce the Generalized Method of Moments (GMM) estimator using the moment conditions from both (a) and (b).

d) Please describe the potential gain and/or loss when you compare the GMM estimator in (c) to either the OLS in (a) or IV in (b).

e) Please describe the potential gain and/or loss when you compare the GMM estimator in (c) to a Maximum Likelihood Estimator (MLE) if the *x* is truly exogenous, and the true distribution of *y* is know and utilized in MLE.

Question 3 (30 minutes)

Consider a CLRM of the following form, at the observation level (dropping individual-level subscripts for simplicity):

$$\begin{aligned} &y = \beta_1 + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_1 * x_2 + \epsilon, \quad \text{where} \\ &\epsilon \sim i.i.d.\left(0, \sigma^2\right) \end{aligned} \tag{1}$$

Part (a)
You are primarily interested in the marginal effect of x_1 and x_2 on the outcome variable, i.e. $\left(\frac{\partial y}{\partial x_1}\right)$, and $\left(\frac{\partial y}{\partial x_2}\right)$. Show the explicit form of these marginal effects, for a given x_1 and x_2.

Part (b)
Let $E\left(x_1\right) = \mu_1$ and $E\left(x_2\right) = \mu_2$ be the population means of the two explanatory variables. Derive the expectation, over, respectively, x_1 and x_2, of the marginal effects you obtained in the preceding part. Let the solutions be labeled as γ_1 and γ_2, respectively.

Part (c)
Using the results form part (b), solve for β_2 and β_3, then insert the resulting expressions into equation (1) in lieu of β_2 and β_3.

After some manipulation, this should produce the following "reduced-form" model:

$$y = \gamma_0 + \gamma_1 x_1 + \gamma_2 x_2 + \beta_4\left(x_1 - \mu_1\right) * \left(x_2 - \mu_2\right) + \epsilon, \tag{2}$$

Show the exact form of the new intercept γ_0.

Part (d)
Now suppose you collect data and estimate both models. To add some realism, let y be weight change for a given individual after enrolling in a multi-week exercise and nutrition program. Let x_1 be hours spent exercising, and x_2 be food consumed, measured in total calories, over the same time period.

What would be the interpretation of β_2 in equation (1) and γ_1 in equation (2), respectively? Which one do you think is more useful and why?

How about the interpretation of β_3 and γ_2?
(Hint: Think about the implied value of the other variable when contemplating the interpretation of a given coefficient)