Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, November, 2018

Part 1: Microeconomics

3 Questions, 2 pages

Note: The minutes assigned to each question indicate the weight given to the question. For example, question 1 is 50 minutes out of a total of 180 minutes and thus counts for 5/18 of the grade for this exam.

Problem 1 (50 minutes)
Suppose that there are two types of consumers (H and L) for a firm's product. The proportion of consumers of type L is $\lambda \in (0,1)$. A consumer of type $i \in \{H, L\}$ enjoys the utility $u_i(x,T) = \theta_i v(x) - T$ when consuming the quantity $x \in [0,1]$ of the good and paying a total amount of T for it, where

$$v(x) = \frac{1-(1-x)^2}{2}.$$

The firm is the sole producer of this good; and its cost per unit is c, with $0 < c < \theta_L < \theta_H$.

(a) Assuming a linear tariff ($T = px$), determine the optimal price p charged by a non-discriminating monopolist. Under which conditions will the monopolist choose to exclude the consumers of type L?

(b) Consider a monopolist that can distinguish the two types (using some observable characteristic) but may only charge a linear tariff to each type ($T_i = p_i x_i$, for $i = H, L$). Characterize the optimal prices p_H and p_L.

(c) Compute the fully optimal nonlinear tariffs and the corresponding quantities. Interpret your results.

Problem 2 (40 minutes)
Suppose that consumer's preference relation $\succsim$ on $X = \mathbb{R}^2_{++}$ is represented by the following utility function:

$$u(x_1, x_2) = u(x_1) + v(x_2),$$

where $u, v : \mathbb{R}_+ \to \mathbb{R}_+$ are strictly increasing, twice-differentiable, strictly concave functions. Assume that $p_1, p_2, m > 0$.

- Show that the Walrasian demand correspondence $x(\mathbf{p}, m)$ is homogeneous of degree zero.
- Show that Walras' law is satisfied.
- Can one prove that goods are normal? Explain!
- Can one prove that the (uncompensated) Law of Demand is satisfied? Explain!

Problem 3 (40 minutes)
Consider a finite normal-form game $\Gamma = \langle N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N}\rangle$. Suppose $\overline{\sigma}_i$ is a dominant strategy and $\underline{\sigma}_i$ is a dominated strategy for player $i \in N$.

- Is $\alpha\overline{\sigma}_i + (1-\alpha)\underline{\sigma}_i$ a dominated or dominant strategy when $\alpha \in (0,1)$?
- Show that $\overline{\sigma}_i$ is a pure strategy.
- Show that there is no belief $\mu \in \Delta(S_{-i})$ for player i such that $\underline{\sigma}_i \in \beta_i(\mu)$, where $\beta_i(\mu)$ is the set of all best response strategies to μ; i.e., $\beta_i(\mu) = \arg\max_{\tilde{\sigma}_i\in\Delta(S_i)} \mathbb{E}_\mu u(\tilde{\sigma}_i, s_{-i})$.

Problem 4 (50 minutes)
Consider a quantity-setting duopoly with inverse market demand function

$$P(q) = \begin{cases} 1-q & \text{for } q \in [0,1]; \\ 0 & \text{for } q > 1 \end{cases}$$

where $q = q_1 + q_2$ is industry output.
The two firms play a two-stage game. At the first stage, firm 1 chooses its output $q_1 \geq 0$. It incurs costs $C_1(q_1) = q_1^2$.
At the second stage, firm 2 chooses its output $q_2 \geq 0$. It incurs costs $C_2(q_2) = q_2^2$.

Firm i's profit is

$$\Pi_i(q_1, q_2) = P(q_1 + q_2)q_i - q_i^2.$$

Now let $q_0 \in (0, 3/7)$ and consider the following pair of strategies:

$$q_1 = q_0;\ q_2(q_1) = \begin{cases} \frac{1}{4}(1-q_0) & \text{if } q_1 = q_0; \\ \\ 1 & \text{if } q_1 \neq q_0. \end{cases}$$

(a) Show that this pair of strategies constitutes a Nash equilibrium of the two-stage game for any choice of $q_0 \in (0, 3/7)$.

(b) Are any of these Nash equilibria subgame perfect? Explain!

(c) Determine the pairs of strategies that are subgame perfect equilibria.

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Part 2: Econometrics and Macroeconomics

Part 2A: Econometrics (3 Questions on 3 pages, 20 minutes each)
Part 2B: Macroeconomics (1 hour, 1 Questions on 2 pages)

Question 1 (20 minutes):

1. (a) Explain what an estimator is and why its optimality can only be assessed via its sampling distribution.

(b) Explain briefly the following properties of an estimator:
(i) *weak consistency*, (ii) *strong consistency*, (iii) *unbiasedness, (iv) full efficiency.*
Hint: if it helps, illustrate your answer in the context of a simple Bernoulli model:

$$X_t \backsim \text{BerIID}(\theta, \theta(1-\theta)), \;\; t{=}1, 2, ..., n, ... \tag{1}$$

(c) In the context of this model, discuss whether the following functions constitute possible estimators of θ:

(a) $\widehat{\theta}_1{=}X_n$, (b) $\widehat{\theta}_2{=}\frac{1}{2}(X_1 - X_n)$, (c) $\widehat{\theta}_3{=}\frac{1}{3}(X_1{+}X_2{+}X_n)$,
(d) $\widehat{\theta}_{n-1}{=}\frac{1}{n-1}\sum_{i=1}^{n} X_i$, (e) $\widehat{\theta}_n{=}\frac{1}{n}\sum_{i=1}^{n} X_i$, (f) $\widehat{\theta}_{n+1}{=}\frac{1}{n+1}\sum_{i=1}^{n} X_i$,

(d) Using your answer in (c): (i) state which of the properties (i)-(iv) hold for the different estimators and select the most optimal.

(ii) Explain why relative efficiency is practically useless when the estimator is inconsistent.

(e) For the simple Bernoulli model derive the Maximum Likelihood Estimator $\widehat{\theta}_{MLE}$ of θ and state its finite sampling distribution.

(f) Explain why the following definition of the Mean Square Error:

$$\text{MSE}(\widehat{\theta}){=}E(\widehat{\theta} - \theta)^2{=}Var(\widehat{\theta}) + [Bias(\widehat{\theta})]^2, \quad \text{for all } \theta{\in}\Theta, \tag{2}$$

makes no sense in the context of frequentist estimation because of the quantifier for all $\theta{\in}\Theta$.

Question 2 (suggested time: 20 minutes)

Consider the following linear regression model for observation i:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 s_i + \mathbf{x}_i' \gamma + \epsilon_i \quad \text{with} \\ \epsilon_i &\sim n\left(0, \sigma^2\right), \quad \forall i = 1 \dots n, \end{aligned} \tag{1}$$

where y_i is the sales price of a single-family residential home (in dollars), s_i is square footage, $\mathbf{x}_i$ includes a set of additional (exogenous) regressors, and ϵ_i is a typical error term with the usual CLRM properties, as shown in the second line of (1).

Part (a), 10 points

(a) What does this model imply for the distribution of y_i, and how could this lead to "practical" problems in the current example?

(b) As given, show $E\left(y_i | s_i, \mathbf{x}_i\right)$, where $E\left(.\right)$ is the expectation operator.

(c) What is the interpretation of β_1 with respect to y_i? Provide mathematical support for your answer.

(d) If one were to use $ln\, y_i$, where ln is the natural logarithm, instead of y_i in (1), how would that change the interpretation of β_1 with respect to y_i? Provide mathematical support for your answer.

(e) If, *in addition*, one were to use the log of square footage, $ln\, s_i$ instead of s_i in (1), how would that change the interpretation of β_1 with respect to y_i? Provide mathematical support for your answer.

Part (b), 6 points

Now consider another model that uses price divided by square footage as the dependent variable, i.e.:

$$\begin{aligned} y_i^* &= \frac{y_i}{s_i} = \beta_0 + \beta_1 s_i + \mathbf{x}_i' \gamma + \epsilon_i \quad \text{with} \\ \epsilon_i &\sim n\left(0, \sigma^2\right), \quad \forall i = 1 \dots n, \end{aligned} \tag{2}$$

(a) What is the new interpretation of β_1? Assuming diminishing marginal utility of housing space holds for the entire range of square footage found in the data, what would you expect its sign to be?

(b) Compute the direct effect of s_i on y_i for this model. How is it fundamentally different from all other effects of square footage on price derived in part (a) above?

(c) At what value of s_i (which may or may not be represented by the data) is this direct effect maximized? Under which additional condition is this indeed a maximum, and how does your answer relate to your argument regarding the expected sign of β_1 from above?

3,

Part (c), 4 points

Somebody suggests using the following mathematically equivalent model to (2) and estimating it via OLS:

$$y_i = s_i\beta_0 + \beta_1 {s_i}^2 + s_i\mathbf{x}_i'\gamma + s_i\epsilon_i \tag{3}$$

(a) How does this model violate CLRM assumptions?

(b) How could this be addressed econometrically to derive consistent estimates for *all* parameters? Show as much mathematical detail as possible.

Question 3 (suggested time: 20 minutes)

Consider the following true relationship between nitrogen fertilizer (N) and yield (y) for a specific crop, for plot i:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 N_i + \epsilon_i \quad \text{if} \quad N_i < N^* \\ y_i &= P + \epsilon_i \quad \text{otherwise,} \quad \text{and} \\ \epsilon_i &\sim n\left(0, \sigma^2\right), \quad \forall i = 1 \ldots n, \end{aligned} \tag{1}$$

where P is often referred to as "plateau yield," and ϵ_i is a mean-zero normal error with equal variance σ^2 for all i, as shown in the second line of (1). N^* is the amount of fertilizer beyond which yield will simply get "stuck" at the plateau. Throughout this question assume that in an actual application N_i goes from zero to 200, and that N^*, while unknown, is located somewhere towards the middle of this range. Also assume no actual N_i exactly equals N^*.

Further assume $\beta_0 > 0$, $\beta_1 > 0$.

Part (a), 8 points

(a) Show $E\left(y_i|N_i\right)$ for both $N_i < N^*$, and $N_i > N^*$, respectively.

(b) Graph $E\left(y_i|N_i\right)$ for the entire range of N_i, with yield on the y-axis, and nitrogen on the x-axis. Add a few scattered dots around this line to symbolize the actual data points.

Part (b), 6 points

Now assume a researcher is unaware of the true relationship between yield and nitrogen, and simply uses an OLS regression of y_i on a constant and N_i, using the *entire data*, to estimate β_0, β_1, and σ^2.

(a) Add your best guess for the estimated regression line (= predicted values for yield for the entire data range) to your graph.

(b) In which directions will the estimates for β_0 be biased? How about for β_1 and σ^2? *(Verbal answer is sufficient)*

Part (c), 6 points

Now assume the researcher knows the general form of the true relationship in (1) as well as N^*.

(a) How could she use the subset of observations with $N_i < N^*$ and basic OLS to predict plateau yield P? Show some math.

(b) How would one derive a standard error for this prediction? Show some math. Assume that the estimated variance-covariance matrix for $\hat{\beta} = \left[\hat{\beta}_0 \quad \hat{\beta}_1\right]'$ is given as $\hat{V}_{\beta}$.

Macroeconomics
1 question, 1 hour

[1] Each individual lives for two periods and thus at each time t the economy is composed of the generation t-1 old people and the generation t young people. Population is constant and is normalized at N=1. Each young person receives an endowment of y widgets in the first period of life and wishes to consume in each period.

Output is non-storable and each young person can save only by acquiring money in the first period of life and carrying it over into old age.

The government produces a paper money which has value if everyone assumes it does. The time t money supply is in the possession of the time t old generation (the t-1 young). The nominal (dollar) money supply is fixed at M. Let P_t be the time t dollar price of output. The time t young assume $P_{t+1} < \infty$ and thus each assumes it is worthwhile to save by accumulating money in the first period of life, to be used to buy goods in the second period.

Let m_t^d be the nominal demand for money by a generation t young individual. The young person at time t solves the following utility maximization problem:

$$\max_{c_1, c_2} \ln c_{1t} + \ln c_{2t}$$

$$\text{s.t. (1) } c_{1t} + m_t^d / P_t = y \text{; (2) } c_{2t} = m_t^d / P_{t+1}^e.$$

In this problem P_{t+1}^e is the time t expected value of P_{t+1}. To simplify the problem, assume individuals have perfect foresight and thus $P_{t+1}^e = P_{t+1}$.

(a) Determine optimal values for c_{it} and m_t^d as functions of y, P_t, and P_{t+1}.

(b) In equilibrium the price level will adjust so that the total demand for money equals the total supply of money (in the possession of the old from generation t-1). Find the equilibrium sequence for the price level starting at t=1.

(c) Suppose that at time 1 there is an x% increase in the money supply (M increases by x% via a cash grant from the government to the time 1 old.). What happens to the sequence of equilibrium prices?

(d) What is the real *gross* return from saving in this model? That is, 1 unit of output saved at time t in the form of money will yield how much output in time t+1?

(e) Suppose there is loans market (or equivalently a bond market) at each time t. What is the equilibrium real interest rate in this market?

Now assume endowments are growing, but the money supply is constant. To be specific, assume each individual in the young t+1 generation has an endowment of $y_{t+1} = y_t(1+g)$, $g>0$. Answer (f)-(h).

(f) Find the sequence of equilibrium prices starting at t =1.

(g) What is the real *gross* return to saving? Explain.

(h) Suppose there is loans market (or equivalently a bond market) at each time t. What is the equilibrium real interest rate in this market?