Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam March 2014

PART I

March 24, 2014

Please answer all 4 questions.
Notice the time allotted to each question.

Problem 1 (50 minutes)

A pure exchange economy $\mathcal{E} = \{(X^i, \succsim^i, \omega^i)_{i=1}^I\}$ with two consumers $i \in \{1, 2\}$ and two commodities $l \in \{1, 2\}$ is considered. The initial endowments are given by $\omega^1 = (0, 8)$ and $\omega^2 = (8, 0)$. Consumer i chooses among commodity bundles in $X^i = \mathbb{R}^2_+$ according to preferences $\succsim^i$ represented by the respective utility functions:

$$u^1(x_1^1, x_2^1) = x_1^1 + \sqrt{x_2^1} \qquad \text{and} \qquad u^2(x_1^2, x_2^2) = x_1^2 + \sqrt{x_2^2}.$$

Let $p \in \mathbb{R}^2_+$, $p \neq 0$, be a price vector.

(a) Draw the Edgeworth box and determine the set of all Pareto optimal allocations.

(b) Determine i's demand correspondence $x^i(p, \omega^i)$.

(c) Provide the definition of Walrasian equilibrium (as exactly as possible).

(d) Determine the Walrasian equilibrium for the exchange economy $\mathcal{E}$.

(e) Determine the Pareto-optimal allocations that can be supported by a price equilibrium with transfers (PET). For this, determine the price vector p, the distribution of wealths m^1 and m^2, and transfers T^1 and T^2.

Consider the exchange economy described above except that preferences of consumer 2 are represented by the following utility function: $u^2(x_1^2, x_2^2) = x_1^2$.

(f) For the new economy, examine which of the equilibrium concepts exists: Walrasian equilibrium, price equilibrium with transfers, or quasi-price equilibrium with transfers?

Problem 2 (50 minutes)

A crime is observed by a group of $n > 1$ people. Each person would like the police to be informed but prefers that someone else make the phone call. Precisely, suppose that each person attaches the value v to the police being informed and bears the psychological cost c if she makes the phone call, where $v > c > 0$. In case nobody calls the police, each of the n people has a payoff of 0. The situation can be modeled as a strategic game in normal form where each of the n players chooses between the two actions $\{Call, Don't\ call\}$.

(a) Find all the pure strategy Nash equilibria of this game and argue that none of them is symmetric.

(b) We are interested in finding the symmetric mixed-strategy Nash equilibrium. Let p be the probability with which each person calls in this symmetric profile.

(b.1) Assuming a symmetric Nash equilibrium, determine the value of p.

(b.2) Consider the two events: "person 1 does not call the police" and "nobody calls the police". Given the symmetric Nash equilibrium found above, determine the respective probabilities of these two events. How do these two probabilities change as n increases?

Examine now the variant where there are two groups of witnesses (e.g., students and teachers in a schoolyard). Every person still attaches the value v to the police being informed of the crime; and each witness of group $i = 1, 2$ bears the cost c_i if she makes the phone call (with $v > c_1 > c_2 > 0$). Assume that the number of witnesses in group i is $n_i \geq 1$ (with $n = n_1 + n_2$).

(c) Find a Nash equilibrium where all of these n witnesses assign a positive probability to each of the two actions $\{Call, Don't\ call\}$.

Problem 3 (40 min)

Solve a consumer's utility maximization problem with three goods with quantities x_0, x_1, x_2, prices $p_0 > 0, p_1 > 0, p_2 > 0$ and income $\theta > 0$.
The specific problem is

$$\max\ x_0^{1/3} \cdot (\min\{x_1, 5x_2\})^{2/3} \quad \text{subject to } p_0x_0 + p_1x_1 + p_2x_2 = \theta.$$

HINT. You can solve this problem in two steps, combining goods 1 and 2 into a composite commodity.

Step 1. Solve the problem

$$\max_{(x_1,x_2)} \min\{x_1, 5x_2\} \quad \text{subject to } p_1x_1 + p_2x_2 = y$$

with solution $z_1(p_1, p_2, y), z_2(p_1, p_2, y)$ and indirect utility $V(p_1, p_2, y)$.

Step 2. Solve the problem

$$\max_{(x_0,y)} x_0^{1/3} \cdot V(p_1, p_2, y)^{2/3} \quad \text{subject to } p_0x_0 + y = \theta.$$

with solution x_0^*, y^*. Then the solution to the original problem is $x_0^*, x_1^* = z_1(p_1, p_2, y^*), x_2^* = z_2(p_1, p_2, y^*)$.

Problem 4 (40 min)

Consider an imperfectly competitive market with $n > 1$ firms $i = 1, \ldots, n$. There are also n goods labeled $i = 1, \ldots, n$ with quantities $q_i \geq 0$.

Firm i produces only good i at marginal cost $c_i \geq 0$ and charges price p_i. Each firm faces a market demand

$$q_i = \begin{cases} A - P & \text{if } P \leq A, \\ 0 & \text{if } P > A \end{cases}$$

with $A > 0$ and $P = \sum_{j=1}^n p_j$. (Such a market demand can occur in an industry that supplies consumers with Leontief utility functions and producers with Leontief production functions.)

Further assume that $A > C$ where $C = \sum_{j=1}^n c_j$.

Determine the Bertrand equilibrium of that industry!

Question I (30 min.)

Consider the simple bivariate classical linear regression model

$$y_i = \alpha + \beta x_i + u_i, \tag{1}$$

with $u_i \sim niid\left(0, \sigma^2\right)$, $i = 1 \ldots 50$ and x_i fixed in repeated sampling.

Part (a)
Suppose that 99 new samples on $y_1 \ldots y_{50}$ are drawn, all with the same 50 values for the explanatory variable, bringing the total sample size to 5,000.

1. Is this what is meant by the assumption that "x_i is fixed in repeated samples" in the original model?
2. Are $x_1 \ldots x_{5000}$ now fixed in repeated samples?
3. How does the idea of re-sampling in this way relate to estimator consistency?
4. What impact could continuing to increase the sample length in this matter have on the OLS estimators of α and β, the usual estimator of σ^2, and on the actual sampling variances for the OLS estimators of α and β?

Part (b)
Now assume instead that the additional data on the dependent variable are obtained with $x_{51} \ldots x_N$ all equal to x_{50} for $N = 5,000$ and beyond.

1. How does this "thought experiment" relate to the assumption that "x_i is fixed in repeated samples" in the original model?
2. What happens to the sampling variances for the OLS estimators of α and β as the sample data increases in this manner?
3. Are these estimators consistent and unbiased?

Part (c)
Now instead assume that the x_i are randomly generated, independent from each other and from all of the model error terms, and the sample size is again increased from 50 to 5,000 (and beyond). Are the OLS estimators for α and β still consistent and unbiased?

Part (d)
Now instead assume that the x_i are randomly generated *from a highly non-Gaussian (i.e. non-normal) distribution*, but remain independent from each other and from all of the model error terms. The sample size is again increased from 50 to 5,000 (and beyond). How, if at all, would your answer to part (c) change?

Question II (30 min.)

Part (a)

Consider a binary response model that is generated from an underlying latent variable model: Person i's response will be observed as $y_i = 1$ if the latent variable $y_i^* > 0$, and as $y_i = 0$ otherwise. The latent variable, in turn, is modeled as

$$y_i^* = \mathbf{x}_i'\boldsymbol{\beta} + u_i, \tag{2}$$

where the error term u_i follows a standard normal distribution with pdf $\phi(u)$ and cdf $\Phi(u)$. Derive the probability that $y_i = 1$.

Part (b)

Now assume that the binary response variable follows the Bernoulli distribution with pdf:

$$f(y_i|\mathbf{x}_i) = p_i^{y_i}(1 - p_i)^{1-y_i}, \tag{3}$$

where $p_i = \text{prob}(y_i = 1|\mathbf{x}_i)$.

Write down the likelihood function (LHF) for of a total of n independent observations from the same Bernoulli distribution. Then write down the *log*-LHF that is used in Maximum Likelihood Estimation (MLE).

Part (c)

Now consider a General Methods of Moments (GMM) estimator based on the result that $E(y_i|\mathbf{x}_i) = \Phi(\mathbf{x}_i'\boldsymbol{\beta})$, which suggests the orthogonality conditions $E[(y_i - \Phi(\mathbf{x}_i'\boldsymbol{\beta}))\mathbf{x}_i] = 0$. Construct a GMM estimator based on this moment condition. Please be specific on what would be the ideal weight used in the construction of the GMM estimator.

Part (d)

Please discuss the pros and cons of MLE and GMM estimators of the probit model described in parts (1) and (3).

Question III (30 min.)

Let y be some measure of a worker's productivity, and let $\mathbf{x}$ denote observable worker characteristics. Furthermore, let w be a binary indicator that takes the value of 1 if a worker has completed a specific training program (i.e. received the "treatment"), and 0 otherwise.

Denote y_0 as the outcome in absence of treatment, and y_1 as the outcome with treatment. Observed and counterfactual outcomes, conditional on $\mathbf{x}$, can then be defined as follows:

$$\begin{array}{ll} y_0|\mathbf{x}, w = 0 & \text{Observed outcome for sub-population of untreated} \\ y_1|\mathbf{x}, w = 1 & \text{Observed outcome for sub-population of treated} \\ y_1|\mathbf{x}, w = 0 & \text{Counterfactual outcome for sub-population of untreated} \\ y_0|\mathbf{x}, w = 1 & \text{Counterfactual outcome for sub-population of treated} \end{array} \tag{4}$$

Part (a)
Describe, in mathematical notation and in words, the *conditional mean independence assumption* and the *overlap assumption*, and assume both hold throughout this exercise.

Part (b)
Under these assumptions, show that the Average Treatment Effect (ATE) is equal to the Average Treatment Effect on the Treated (ATT), conditional on $\mathbf{x}$.

Part (c)
Under these assumptions, show that

$$E(y|\mathbf{x}, w = j) = E(y_j|\mathbf{x}), \quad j = 1, 2 \tag{5}$$

Hint: Recall the general relationship $y = y_0 + w(y_1 - y_0)$. In words, what does this result imply?

Part (d)
Assume $E(y|\mathbf{x}, w = j)$ can be written as a regression function $m_j(\mathbf{x}) = \alpha_j + \mathbf{x}'\boldsymbol{\beta}_j, \quad j = 0, 1$. Express the *conditional-on-*$\mathbf{x}$ ATE for the sub-population of individuals with characteristics $\mathbf{x}$ as a function of the regression parameters given above. You can label the conditional ATE as $\tau_{ate}(\mathbf{x})$.

In generic terms, show the expression for the *un*conditional ATE (i.e. relaxing conditionality on $\mathbf{x}$). Label this term τ_{ate}. What would you need to know about $\mathbf{x}$ to actually solve for it?

Part (e)
Consider a sample of n workers, of whom n_0 did not receive the treatment, and n_1 did receive the treatment. Assume there exist consistent estimators for $m_j(\mathbf{x}), \quad j = 0, 1$ - label them with a "hat" - symbol.

In words, describe how you would derive a consistent *regression adjustment estimator* for τ_{ate}.

Macro Question

[1] Consider the following growth model:

Total time t output is $Y_t = A\, K_t^{\alpha} \hat{L}_t^{1-\alpha}$,

in which K_t is the time t aggregate capital stock and $\hat{L}_t$ is total hours worked.

Individuals live for two periods, working in the first period of life and consuming in the second (last) period of life. Consider, for example, time t. At time t there are L_t members of generation t (the time t young) and L_{t-1} members of the t-1 generation (the time t old). The population growth rate is g, so $L_t = L_{t-1}(1+g)$.

The young individuals work for θ hours, receiving the real wage w_t per unit of labor (w_t, being the marginal product of labor, MPL_t). Total hours are thus $\hat{L}_t = \theta\, L_t$. The time t generation will convert their wages into time t+1 capital and thus $K_{t+1} = \theta w_t L_t$. Members of the time t generation rent their capital, receiving [$\theta\, w_t \times MPK_{t+1}$] units of t+1 consumption (MPK_{t+1} being the time t+1 marginal product of capital).

(a) Write the expressions for the MPL_t and the MPK_t.

(b) Let $k_{t+1} \equiv K_{t+1} / \hat{L}_{t+1}$. Find the steady state value for k. Call it k_{ss}.

(c) What is steady state consumption for the old at any time t?

(d) Suppose there is a decrease in θ, hours worked per young person. What is the impact on k_{ss}? What is impact on steady state consumption in old age?

(e) Suppose there is a decrease in g. What is the impact on k_{ss}, the steady state real wage, and the steady state marginal product of capital?

(f) Given the decrease in g, what is the impact on steady state consumption in old age. Under what conditions will it decrease? Explain the intuition behind your answer. Suppose α =1/2. Will there be an increase or decrease in steady state old-age consumption?

Macro Qualifying exam question (Spring 2014)

Suppose suppliers are locating in a large number of scattered, competitive markets, and demand for goods in each period is distributed unevenly over the markets. There are both general and relative price movements. Aggregate demand is $y_t + P_t = x_t$, where x_t is a exogenous policy variable, and its change has a distribution $\Delta x_t \sim N(0, \sigma^2)$.

Quantity supplied in each market z is written as:

$y_t(z) = \gamma[P_t(z) - E(P_t|I_t(z))], \qquad \gamma > 0$

where z is an index for the market, $P_t(z)$ is the actual price in market z at time t, and $E(P_t|I_t(z))$ is the expected current, general price level, conditioned on information available in market z at time t, $I_t(z)$.

Information $I_t(z)$ contains a prior distribution $P_t \sim N(\bar{P}_t, \delta^2)$ (which is based on past data), and it also contains the observed, local price $P_t(z)$, which is equal to the unobserved general price level plus a shock:

$P_t(z) = P_t + z$

where $z \sim N(0, \tau^2)$ and it is independent of P_t. All variables are in log.

a) What is $E(P_t|I_t(z))$? Write down and explain your answer in terms of $\tau^2, \delta^2, P_t(z)$ and $\bar{P}_t$.

b) Derive the aggregate supply y_t in terms of $\tau^2, \delta^2, \gamma, P_t$ and $\bar{P}_t$.

c) Solve for the equilibrium output and the equilibrium inflation in terms of τ^2, δ^2 and change of the shift variable Δx_t (and may be also its lag). [Hint: guess a solution for P_t.]

d) Based on your results, briefly explain the following statement: "any attempt to exploit the Phillips curve and permanently increase employment by systematically creating higher inflation would be futile and only give rise to higher inflation."