Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam

Fall 2006

PART I

October 17, 2006

Please answer all 4 questions. Notice the time allotted to each question.

Problem # 1 (45 minutes)

Each of two consumers has an income of $m = 100$. Let X be the amount of money spent on food and Y the amount of money spent on all other goods. Obviously, the prices are $p_x = 1$ and $p_y = 1$. The preferences of the first consumer can be represented by the utility function $u(x,y) = xy$ and those of the second consumer can be represented by the utility function $v(x,y) = xy^2$.

The government plans to implement a food stamp program and is considering two somewhat different variations.

(A) Under the first plan each participant will be able to purchase from the government up to $50 worth of food stamps at a price of 50 cents for $1 worth of stamps. However, no participant in the program will be allowed to sell any of the stamps he purchased from the government.

(B) The second plan is the same as the first, except that participants will be allowed to sell the food stamps they bought from the government to whoever they wish (including people who do not participate in the program). Furthermore, the government expects that after this plan is implemented, food stamps will be bought and sold in the market at a price of 75 cents for $1 worth of stamps.

Assume that each of these consumers can participate in the food stamp program.

(a) How will each of them allocate his income if the first plan is implemented?

(b) How will each allocate his income if the second plan is implemented?

(c) Suppose before the government decides which plan to implement it asks each consumer which of the two plans he prefers. What answer will each of them give?

Problem # 2 (45 minutes)

A profit maximizing firm sells its homogeneous output in two markets which are completely separated from one another. The inverse demand it is facing in the two markets are

$$p_1 = 50 - q_1$$

$$p_2 = 35 - .5q_2,$$

where p_i and q_i are the price and quantity in market i respectively, $i = 1, 2$. The firm has no fixed cost. Due to different transportation cost its marginal cost for quantities sold in the two markets are $MC_1(q_1) = 5$ and $MC_2(q_2) = 10$.

(a) Derive the prices the firm would charge and the quantities it would sell in each market.

(b) Now suppose the government issues a regulation according to which the firm has to charge the same price in both markets. What price would it charge? How much will it sell in each market?

(c) Compare the aggregate consumer surplus, the producer surplus and total surplus in case (a) to those in case (b). If you were asked whether the governmental regulation in (b) is desirable or not, what will be your answer?

Problem # 3 (45 minutes)

Consider an exchange economy with two agents and two goods. The preferences of agent 1 can be represented by the utility function

$$u_1(x_{11}, x_{12}) = -(x_{11} - 3)^2 - (x_{12} - 3)^2$$

and those of agent 2 can be represented by the utility function

$$u_2(x_{21}, x_{22}) = x_{21} + x_{22}$$

where x_{ij} is the quantity that agent i has of good j, $i,j = 1,2$.

(a) Given that the total initial endowment is $\omega = (10, 8)$, identify all Pareto efficient allocations. (Hint: You might want to use a diagram).

(b) Suppose now that the individual initial endowments are $\omega_1 = (6, 0)$ and $\omega_2 = (4, 8)$. Derive a competitive allocation for this economy. Is it unique?

Problem # 4 (45 minutes)

The government of Kenya (GOK) develops a new higher yielding maize variety for public release. GOK hires you to estimate how the variety will benefit country consumers and producers.

(1) Outline the framework you would use to estimate the economic benefits of the innovation. Make sure to detail the impact of the innovation on the supply and demand for maize.

(2) Will producers always benefit from the innovation?

(3) Will consumers always benefit from the innovation?

(4) What information would you require to estimate the demand equation for maize in Kenya? In particular:

(4.a) Specify the type of data you would collect and define each variable that you would include in the demand equation.

(4.b) Specify the functional form of the equation and discuss why you chose it over alternative forms.

(4.c) Discuss the basic problems you anticipate having to deal with in the estimation and how you would handle them.

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PART II

October 20, 2006

Please answer all 3 questions. Notice the time allotted to each question.

Problem # 1 (60 minutes)

Consider the multiple regression model

$$Y_i = \sum_{j=1}^{k} \beta_j X_{i,j} + \varepsilon_i .$$

Suppose that an unbiased estimate of

$$\frac{\partial E[Y_i \mid X_{i,1},\ldots,X_{i,k}]}{\partial X_{i,j}}$$

is desired.

(1) Under what circumstances will the least squares estimator of β_j produce such an estimate? (I.e., what must be assumed about the way the data were generated in order for the least squares estimator of β_j to be unbiased?) Show your work. You may use the $Y = X\beta + \varepsilon$ form of the model if you prefer.

(2) What simple specification advice does this result suggest?

(3) Briefly comment on why an unbiased estimator (or an estimator with known bias) is essential for constructing hypothesis tests and confidence intervals.

(4) Briefly comment on what circumstances might cause one to prefer a biased estimator.

Problem # 2 (60 minutes)

Consider the following **linear structural model:**

$$y_t = \alpha^T \mathbf{X}_t + \varepsilon_t,\ \varepsilon_t \sim \mathrm{N}(0, \sigma^2),\ t \in \mathbf{T} := \{1, 2, \ldots, T, \ldots\}, \qquad \#$$

where the $m+1$ vector of explanatory variables $\mathbf{X_t}$ is assumed to be stochastic and correlated with the error term, in the sense that:

$$\text{(i)} \qquad E(\mathbf{X}_t \varepsilon_t) = \mathbf{d} \neq \mathbf{0}. \qquad \#$$

(a) Explain how (i) is explained in the traditional textbook approach and why such an explanation renders condition (ii) $E(\mathbf{Z}_t \varepsilon_t) = 0$ below problematic.

(b) Explain why under (2) the OLS estimator of α :

$$\hat{\boldsymbol{\alpha}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}, \qquad \#$$

$\sigma$is **biased** and **Inconsistent** unless $\mathbf{P}_{T\to\infty} \lim(\frac{d}{T}) = 0$.

(c) Discuss critically how the traditional textbook approach 'solves' this problem using the **instrumental variables (IV)** $\mathbf{Z}_t$, a $p \times 1$ vector ($p \geq m$), such that

$$\text{(ii)} \qquad E(\mathbf{Z}_t \varepsilon_t) = 0$$
$$\text{(iii)} \qquad E(\mathbf{X}_t \mathbf{Z}_t^T) = \Sigma_{23} \neq 0$$
$$\text{(iv)} \qquad E(\mathbf{Z}_t \mathbf{Z}_t^T) = \Sigma_{33} > 0.$$

Giving rise to a much better estimator:

$$\hat{\boldsymbol{\alpha}}_{IV} = (\mathbf{X}^T\mathbf{P}_Z\mathbf{X})^{-1}\mathbf{X}^T\mathbf{P}_z\mathbf{y}, \qquad \#$$

where $\mathbf{P}_Z = \mathbf{Z}(\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T$.

(d) Explain how the **IV** estimator (4) is derived, and why for its consistency the textbook argument imposes implicitly the additional restriction:

$$\text{(v)} \qquad E(\mathbf{Z}_t \mathbf{y}_t) = \boldsymbol{\sigma}_{31} \neq 0.$$

(e) Give a more satisfactory account as to how one justifies (i)-(v) and why the **IV** estimator is not chosen on the basis of $E(\mathbf{Z}_t \boldsymbol{\sigma}_t) = 0$, but on *statistical adequacy* grounds. Explain how the restrictions (iii) and (v) can be reliably tested.

Problem # 3 (60 minutes)

Consider a two-period economy composed of *identical* individuals. There is no money. However, there is a bond (loans) market that operates at time 1. Denote the time 1 interest rate by R. Let c_t be time t consumption, let l_t be time t work effort and let b_1 be the bonds purchased by an individual at time 1 (positive or negative).

Output at time t is $y_t = f(l_t) = a_t l_t + z_t,\ z_t < a_t$.

Individual utility is

$$U(c_1, l_1, c_2, l_2) = [\ln(c_1) + \ln(1 - l_1)] + \frac{\ln(c_2) + \ln(1 - l_2)}{1 + \theta}$$

which is maximized subject to the lifetime budget constraint

$$c_1 + \frac{c_2}{1 + R} = y_1 + \frac{y_2}{1 + R}.$$

(a) Assume that $a_1 = a_2$ and $z_1 = z_2$. Determine equilibrium values for $R,\ c_t,\ l_t\ (t = 1,2)$ and b_1.

(b) Assume that $a_1 = a_2$ but that $z_1 > z_2$. Determine equilibrium values for $R,\ c_t,\ l_t\ (t = 1,2)$. Compare the interest rate in this to that in (a) and explain the difference.

Now introduce government into the model. Government spending at time t is g_t. Spending is financed with lump sum taxes equal to τ_t, with $\tau_t = g_t$. Assume that for parts (c) - (e) below $a_1 = a_2$ and $z_1 = z_2$.

(c) Assume that $g_1 = g_2 = \overline{g}$. Determine equilibrium values for $R,\ c_t$ and $l_t\ (t = 1,2)$.

(d) Assume that $g_1 > g_2 = \overline{g}$. Determine equilibrium values for $R,\ c_t$ and $l_t\ (t = 1,2)$. Explain the difference between the answer here and in (c).

Deficit spending:

(e) Suppose $g_1 = g_2 = \overline{g}$, but the government sets $\tau_1 = 0$ and finances g_1 by borrowing. The loan and interest are paid off in period 2 so that the new level of lump sum taxes in period 2 is $\widetilde{\tau_2} = g_1(1 + R) + g_2$. What happens to the rate of interest as compared to your answer in (c)? Explain.