Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam

Summer 2006

PART I

June 19, 2006

Please answer all questions. Notice the time allotted to each question.

Problem 1. (45 minutes)

Epidemiologists Have been studying a cohort of over 2,000 female nurses for quite some time; around 35% of these nurses had been diagnosed with breast cancer. Interestingly enough, it has been possible to interview the mothers of most of these nurses and obtain data on the frequency with which the nurse-daughter ate or drank an average serving of each of 30 food items when she was a youngster. (You may ignore any measurement error in these data for purposes of this question). These pre-school food consumption data (and additional information, such as body mass index) were used to estimate a regression model analyzing the impact of pre-school consumption of these foods on the probability of contracting breast cancer later in life. The principal result of this study - which really took place, was published in the *International Journal of Cancer* this year, and which attracted a degree of media attention - was that pre-school consumption of french fries has a significant impact on the incidence of breast cancer as an adult.

Let $\mathbf{Y}_i$ be the breast cancer variable; one if nurse i was diagnosed with breast cancer and zero otherwise. And let $\mathbf{X}_{i,j}$, $j = 1,\ldots,30$ be quantitative measures of the pre-school consumption by nurse i of each of the 30 food items; in particular, $\mathbf{X}_{i,30}$ is the measure of nurse i's pre-school consumption of fries. Let $\mathbf{X}_{i,j}$, $j = 31,\ldots,35$ denote the other explanatory variable values for nurse i, including $\mathbf{X}_{i,35}$ equals to one for all nurses.

1. Would it be appropriate to analyze the impact of french fry consumption on the breast cancer incidence by estimating β_{30} in the regression model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ using OLS? Why or why not?

2. The author of this journal article finds that the coefficient on $\mathbf{X}_{i,30}$ differs from the value it would have if french fry consumption has no impact on breast cancer incidence by 3.4 times its estimated standard of error. Consequently, it is concluded that the null hypothesis (that pre-school french fry consumption has no impact on adult breast cancer incidence) can be rejected. Assuming that all the model assumptions are valid and that the sample is sufficiently large, indicate the significance level at which this null hypothesis can be rejected with a simple sketch.

3. Presuming - as is no doubt the case - that this researcher would have implicated any other food item (such as liver, food item # 28) as a cause of breast cancer had its coefficient been statistically significant, what hypothesis should have been tested? How could this test have been done using the maximum likelihood framework?

Problem 2. (45 minutes)

Answer the following four questions.

A. Explain the Durbin-Watson (D-W) test for first order Autocorrelation in the context of the Linear Regression model

$$y_t = \beta_0 + \beta_1 x_t + u_t,\ u_t \sim \text{NIID}(0, \sigma^2); \tag{1}$$

be explicit about about the null, alternative hypotheses, the test statistic and its rejection region.

B. Compare and contrast the D-W test to the t-test arising from testing the hypothesis:

$$H_0 : \beta_1 = 0, \text{ vs. } H_1 : \beta_1 \neq 0.$$

in the context of (1).

C. Using your answer in B, explain why adopting the alternative in the case of the D-W test is tantamount to committing *a fallacy*. How can one address this problem in practice?

D. Compare and contrast the Dynamic Linear Regression (DLR) specification

$$y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 x_{t-1} + \alpha_3 y_{t-1} + v_t,$$

with the Autocorrelation-Corrected Linear Regression (ACLR) model:

$$y_t = \beta_0 + \beta_1 x_t + u_t,\ u_t = \rho u_{t-1} + \varepsilon_t. \tag{2}$$

Explain why when the common factor restrictions are invalid the OLS estimator $\widehat{\beta} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{y}$ is both biased and inconsistent.

Problem 3. (45 minutes)

Consider an economy composed of finitely many **identical** individuals. Each individual has the instantaneous utility function

$$u(c_t, l_t) = 2c_t^{1/2} - l_t,$$

in which c_t is time t consumption and l_t is time t work effort. Assume also that the production function for each individual is

$$c_t = f(l_t) = al_t,$$

where a is an exogenous constant.

(a) Supposing individuals live for one period, find the optimal c_t and l_t.

(b) Now assume individuals live forever, there is a loan market and the utility function of an individual is

$$U = \sum_{t=1}^{\infty}(u(c_t, l_t)/(1+\theta)^{t-1}),\text{ where } u(c_t, l_t) \text{ is as above and } 0 < \theta < 1.$$

Find equilibrium values for c_t , l_t and the rate of interest r_t, $t = 1, 2, \ldots$. Explain your results.

Now assume that government taxes income directly to finance government spending. Government spending at time t is $g_t = \tau_t y_t = \tau_t(al_t)$, where τ_t is the marginal (and average) tax rate, $0 < \tau_t < 1$.

(c) Suppose that the tax rate is fixed at $\tau_t = \tau$. Find equilibrium values for c_t, l_t and the rate of interest r_t .

(d) Taking into account that work effort at any time t depends upon the tax rate, find tax revenues, R, as a function of τ, $R(\tau) = [\tau al(\tau)]$. What tax rate maximizes government revenues? A graph may be helpful. For a **given** (feasible) level of government spending, what is the efficient tax rate, that is, which tax rate maximizes individual welfare **given** the level of government spending? Explain.

(e) Suppose that at time 1 there is a temporary increase in the tax rate so that $\tau_1 > \tau = \tau_2 = \tau_3 = \ldots$. Find the effect of this increase on r_1 . Explain.

Problem 4. (45 minutes)

On January 1, 2003, the Manhattan, NY City Council banned smoking in all bars. The City Council hires you to estimate the causal effect of the ban on bar sales. They provide you with sales data for 50 randomly chosen Manhattan city bars and 50 randomly chosen Brooklyn (New York - right across the river from Manhattan) bars from 2002 and 2004. Assume that Brooklyn continues to allow smoking in bars during the entire time.

You define the treatment group as bars in Manhattan (subscript = M) and the control group as bars in Brooklyn (subscript = B). The treatment, X={0,1}, is "smoking is banned". Your variable of interest is the expected sales at Manhattan City bars. You have two periods, Y=2002 and Y=2004: Define E_{ban} as the causal effect of the ban on Manhattan City bar sales.

Let $E(S_i^1 \mid Y)$ equal expected sales for bars in city i in year Y if treated (smoking banned). Note that i denotes cities (Manhattan and Brooklyn). Similarly, let $E(S_i^0 \mid Y)$ equal the expected sales for bars in city i in year Y if not treated (smoking not banned).

Please answer the following:

✓ **1.** Which of the following 8 values do you observe in your study?
(A) $E(S_M^1 \mid 2002)$, (B) $E(S_M^1 \mid 2004)$, (C) $E(S_M^0 \mid 2002)$, (D) $E(S_M^0 \mid 2004)$
(E) $E(S_B^1 \mid 2002)$, (F) $E(S_B^1 \mid 2004)$, (G) $E(S_B^0 \mid 2002)$, (H) $E(S_B^0 \mid 2004)$.

2. If you could observe all 8 values from the previous question, what would be the correct estimate of the causal effect, E_{ban}, of the smoking on sales at Manhattan bars in 2004?

3. Given that you cannot implement your estimator in question 2, consider the following two simple alternatives:
(a) $E(S_M^1 \mid 2004) - E(S_M^0 \mid 2002)$
(b) $E(S_M^1 \mid 2004) - E(S_B^0 \mid 2004)$
Under what assumptions will these estimators correctly estimate the causal effect? Discuss the plausibility of those assumptions. Be precise. [Note that parts (a) and (b) do not have the same answer].

4. Propose an alternative estimator of the causal effect of the smoking ban that uses information from both years and both sites. Specify your model completely as well as the statistical test you would run. Under what assumptions would your estimator correctly estimate the treatment effect? What additional data might you want to collect?

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PART II

June 22, 2006

Please answer all questions. Notice the time allotted to each question.

Problem 1. (45 minutes)

Consider a market in which n Cournot oligopolies are operating, all producing the same homogenous product and each deciding on the quantity that it produces. The inverse demand in the market is given by

$$p = 110 - \sum_{j=1}^{n} q_j$$

and the total cost function of each firm is $TC_j(q_j) = c_j q_j$, where $c_j > 0$ for each firm $j = 1, \ldots, n$.

(a) Suppose there are two firms in the market with marginal cost $c_1 = c_2 = 60$. How much will each firm produce in equilibrium? What will be the price in the market?

(b) What are the equilibrium quantities and price if a third firm, with marginal cost $c_3 = 60$, joins the market?

(c) Suppose now that a fourth firm enters the market, with marginal cost $c_4 = 10$. What are the quantities and price in the Cournot equilibrium in this case?

(d) How does the entrance of the third firm affect the welfare? How does the entrance of the fourth firm affect the welfare? Can you draw any conclusions about the relation between the number of firms in this market and the "level of welfare"?

Problem 2. (60 minutes)

Consider an industry with three commodities, denoted 1, 2 and 3, and two firms, Fincastle Corporation (FC) and Galliburton Industries (GI).

FC uses commodity 1 to produce commodity 2. Its technology is given by the production function $f(z_1) = z_1^{1/2}$ for $z_1 \geq 0$.

GI uses commodities 1 and 2 to produce commodity 3. Its technology is given by the production function $g(z_1,z_2) = z_1^{1/3}z_2^{1/3}$ for $z_1 \geq 0,\ z_2 \geq 0$.

Part 1.

Given the price vector $p = (p_1,p_2,p_3) \geq 0$;

(a) Determine the profit maximizing input z_{F1}^* and the corresponding output $q_{F2}^* = f(z_{F1}^*)$ for FC.

(b) Determine the profit maximizing input combination (z_{G1}^*,z_{G2}^*) and the corresponding output $q_{G3}^* = g(z_{G1}^*,z_{G2}^*)$ for GI.

Part 2.

Suppose FC and GI have merged into one firm, Mammoth Manufacturers (MM). MM now uses both technologies so that it produces the maximal quantity of commodity 3, $h(z_1)$, from any given quantity $z_1 \geq 0$ of commodity 1. In particular, it divides any given quantity z_1 into two parts, z_1' and z_1''. The quantity z_1'' is used as an input in the production of commodity 2 using the technology of FC, and z_1' is used directly as an input in the production technology of GI.

(c) Determine $h(z_1)$ by evaluating $g(z_1',f(z_1''))$ at the solution of the problem

$\max g(z_1',f(z_1'')) = (z_1')^{1/3} \cdot (z_1'')^{1/6}$ subject to $z_1' + z_1'' = z_1,\ z_1' \geq 0,\ z_1'' \geq 0$.

HINT: You can look at it as a case of maximization of a Cobb-Douglas utility function.

(d) Assume now that MM can not purchase or sell any quantity of commodity 2. It can, however, purchase and/or sell any quantities of commodities 1 and 3. Given prices $(p_1,p_2,p_3) \geq 0$, find the solution z_{M1}^* of MM's profit maximization problem

$$\max p_3 h(z_1) - p_1 z_1, \text{ subject to } z_1 \geq 0$$

and the corresponding output $q_{M3}^* = h(z_{M1}^*)$.

(e) Answer part (d), but now under the assumption that MM can purchase and/or sell any quantities of commodity 2 it wishes.

Problem 3. (60 minutes)

Consider an exchange economy with two goods, x_1 and x_2, and where **all consumers** have identical preferences represented by the utility function

$$U(x_1,x_2) = -e^{-x_1} - e^{-x_2}.$$

(a) Suppose a consumer can consume positive as well as negative quantities of each of the two goods. Show that the consumer's demand functions $D_i : \mathbf{R}^3_+ \to \mathbf{R}^1$, $i = 1,2$ are

$$D_1(p_1,p_2,I) = \frac{I - p_2\Delta}{p_1 + p_2}$$

$$D_2(p_1,p_2,I) = \frac{I + p_1\Delta}{p_1 + p_2}$$

where

$$\Delta = \log p_1 - \log p_2 \equiv \log\left(\frac{p_1}{p_2}\right).$$

(b) What are the demand functions if quantities demanded can not be negative? (Hint: Be very careful).

(c) The consumers in this exchange economy are of two types. The initial endowment of a consumer of type A is $\varpi_A = (1,0)$ and that of a consumer of type B is $\varpi_B = (0,1)$. (Remember that all preferences are identical and continue to assume that quantities demanded are restricted to be non negative).

(c.1) Derive the competitive (Walrasian) equilibrium, both prices and quantities, when there are two consumers in the economy, one of type A and the other of type B. (Hint: You might want to use the normalization $p_1 + p_2 = 1$).

(c.2) Consider now this economy with one consumer of type A and $k \geq 1$ consumers of type B. Show that in a competitive equilibrium the prices are characterized by the equation

$$\frac{p_1}{p_2}\log\left(\frac{p_1}{p_2}\right) = \frac{k-1}{k+1}.$$

(c.3) Now consider the economy with $k_A \geq 1$ consumers of type A and $k_B \geq 1$ consumers of type B. Under what conditions on k_A and k_B will each consumer consume in an equilibrium only one of the goods?

Problem 4. (30 minutes).

Consider a firm that uses a technology of constant returns to scale. A claim was made that in that case its Average Cost function, $AC(q)$, is a constant function. If you agree with this claim then **prove** it. (Notice that you do not have to provide an algebraic proof. A formal and complete verbal proof will suffice). On the other hand, if you do not agree with this claim then provide a counter example.