**Departments of Economics and of Agricultural and Applied Economics**

**Ph.D. Qualifying Exam**

**Fall 2007**

**PART I**

**October 22, 2007**

**Please answer all 4 questions. Notice the time allotted to each question.**

**Problem 1 (30 minutes)**

Consider a Cournot duopoly in which two firms produce a homogenous product and let $q_i$ be the quantity produced by firm $i$, $i = 1, 2$. The variable cost and marginal cost of each firm are zero. The market demand for the product is $p = 2 - (q_1 + q_2)$.

(a) Derive the Nash equilibrium in this market.

(b) Suppose now that each firm is interested not only in its profits but also in the quantity it sells. In particular, assume that each firm has the following payoff function

$$U_i(q_1, q_2) = (1 - \alpha_i) \cdot \Pi_i(q_1, q_2) + \alpha_i \cdot q_i,$$

where $0 < \alpha_i < 1$. Derive the Nash equilibrium in this market in each of the two following cases and in each case compare it to the Nash equilibrium you have derived in (a) above:

(b1) $\alpha_1 = \alpha_2 > 0$

and

(b2) $\alpha_1 > \alpha_2 > 0$.

**Problem 2 (60 minutes)**

Consider a market for newly developed, differentiated products. We assume that there are $n$ firms, each developing a new product. These products are strictly differentiated in quality and are "competitive" in the sense that they share the same global demand for this type of good. The more time a firm spends on development of its product, the higher the quality of the product of that firm at release time. But the first firm to release its product has an advantage: the customers it obtains at release time will not subsequently switch to the product of one of its rivals. This leads to the following formulation of this form of competition.

Each firm $i$ selects a time of release $t_i \in [0,1]$ for its product. Upon release at time $t$ a firm's product has quality $Q(t) \in [0,1]$, where $Q\colon [0,1] \to [0,1]$ is a continuous and increasing function with $Q(0) = 0$ and $Q(1) = 1$. We assume that firm $i$ captures a fraction $Q(t_i)$ of the remaining market share if it releases its product at time $t_i$. If multiple firms release their products at the same time, they split the obtained market share equally. Hence, firm $i$'s captured market share at time $t_i$ is given by

$$m_i(t) = \frac{Q(t_i)}{\#\{h \in N \mid t_h = t_i\}} \cdot \left( 1 - \sum_{j:\, t_j < t_i} m_j(t_j) \right),$$

where $N = \{1, \ldots, n\}$ is the set of all firms and $t = (t_1, \ldots, t_n)$ is the vector of all release times.

Finally, we assume that each firm tries to capture the largest possible market share.

(a) Formulate this problem as a strategic form game. Be precise in formulating the strategy space for each firm as well as its payoff function.

(b) Suppose that the $n$ firms in the market release their products at times $t_1 \leqq t_2 \leqq \cdots \leqq t_n$. Show the following property:

$$\sum_{i=1}^{n} m_i(t_i) = 1 \text{ if and only if } t_n = 1.$$

Give a proper economic interpretation and discussion of this property.

(c) Let the market be a potential duopoly, i.e., $n = 2$. Find all Nash equilibria in this market by formulating the best responses correspondences for each of the two firms and identifying the Nash equilibria from those.

**Problem 3 (60 minutes)**

Consider a world with 2 commodities. In the republic of Egalia all consumers *i* have the **same endowment bundle** $\omega_i \in \mathbf{R}^2_{++}$. There are two types of consumers in Egalia, called Megas and Nanos.

- A Mega has the Cobb-Douglas utility function $u_i(x_i) = (x_i^1)^{1/2} \cdot (x_i^2)^{1/2}$ for consumption bundles $x_i = (x_i^1, x_i^2) \in \mathbf{R}^2_{++}$.
- A Nano has the Cobb-Douglas utility function $u_i(x_i) = (x_i^1)^{3/4} \cdot (x_i^2)^{1/4}$ for consumption bundles $x_i = (x_i^1, x_i^2) \in \mathbf{R}^2_{++}$.

The following two facts are known:

1. The consumption bundle of a Mega in a competitive equilibrium is $x_i^* = (20, 10)$.
2. 40% of Egalia's population are Megas and 60% are nanos.

(a) Determine the competitive equilibrium consumption of a Nano.

(b) Determine $\omega_i$.

(c) Determine the competitive equilibrium price system of the normalized form $p^* = (1, p_2^*)$.

(d) Suppose the Mega population in Egalia increases because of immigration from the neighboring country of Megalia. Consider the new competitive equilibrium allocation for Egalia's economy: In deriving your answers to the questions below you can use the following known facts

(i) The new normalized equilibrium price system is $(1, \overline{p}_2)$ with $\overline{p}_2 > p_2^*$.

(ii) In a population composed of both Megas and Nanos, the normalized equilibrium price system is always of the form $(1, p_2)$ with $26/21 < p_2 < 26/7$.

(d1) Is a Nano consumer economically better off than before? Prove your answer.

(d2) Is a non-immigrant Mega economically better off than before? Prove your answer.

(e) All inhabitants of Megalia are Megas (no calculations are necessary to answer the following two questions)

(e1) Is a Mega who moves from Megalia to Egalia better off after the emigration?

(e2) How is the per capita equilibrium consumption in Megalia affected by emigration?

**Problem 4 (30 minutes)**

This question has three different parts. Answer each of them separately.

(a) Wine is a heterogeneous product. Higher quality bottles are more expensive. Like most states, Virginia taxes wine based on the quantity sold. In 2007, for example, the tax rate is \$1.51 per gallon of wine. Occasionally, when the tax rate is raised, the average price of wine (calculated as total revenue divided by total quantity sold) rises by more than the tax increase. Assume that firms set price at marginal cost, which varies with quality, market supply for each bottle is constant, consumers' preferences and incomes are unchanged, population is fixed, and no other policy changes occur during the study period. Given these assumptions, can you offer an explanation for why the average price of wine would rise by more than the tax increase?

(b) Salvador Dali, the surrealist painter, died in 1989. Assume that his paintings are sufficiently similar that we can treat them as a homogeneous product. Consider a new tax on the sale of Dali's paintings. Derive an expression for how the change in the price actually received by sellers depends on the market supply and demand elasticities, which you can treat as constant. Who is likely to bear more of the tax burden, buyers or sellers, and why?

(c) There are many examples of price discrimination in which consumers with high income are charged higher prices than those with lower income. Why does high elasticity of demand often accompany low income, and are there any goods for which you would expect the opposite to be true? Use the Slutsky equation to explain your answer.

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**PART II**

**October 25, 2007**

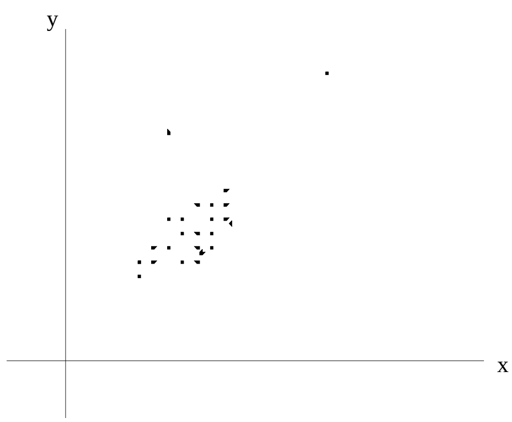
**Please answer all 4 questions. Notice the time allotted to each question.**

**Problem 1 (45 minutes)**

Consider the usual multiple regression model:

$$Y = X\beta + \varepsilon \qquad \varepsilon \sim N(0,\ \sigma^2 I) \qquad X \text{ is a fixed } (n \times k) \text{ with } r(X) = k.$$

and the following scatterplot:




(a) In words, what is the difference between an outlier and a highly influential observation?

Illustrate these concepts, using the scatterplot.

(b) Which of these two concepts is mostly closely related to non-Gaussianity, either in the conditional distribution of $Y$ or in the distribution of $\varepsilon$ ?

(c) How might you in practice diagnostically check for an outlier using a simple hypothesis test which is valid for large $n$?

(d) Keeping in mind that $\hat{\beta}^{ols}$ is a linear function of $Y,$ state an expression for the influence of the $i^{th}$ observation on $\hat{\beta}^{ols}$ , i.e., for $\partial\hat{\beta}^{ols}/\partial Y_i$ , in the multiple regression model. Comment briefly on the desirability (or otherwise) of highly influential observations in a data set.

**Problem 2 (45 minutes)**

(a) State and explain the Gauss-Markov (G-M) theorem as it relates to the optimality of the least-square estimator $\widehat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T y$ of $\beta$ in the context of the linear regression model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}.$$

Hint: state the assumptions and conclusions explicitly.

(b) Discuss the view that the G-M theorem constitutes a 'robustness' result for estimator of $\beta$, by providing optimality without the Normality assumption.

Hint: Explain what the G-M theorem provides in terms of statistical inference propositions *without the normality assumption* and how in practice that information is inadequate for reliable inference purposes.

(c) How are the G-M theorem inference results supplemented in practice by invoking some central limit theorem and what are the weaknesses of such supplements.

(d) Explain the difference between a t-test for the significance of a coefficient, say $\beta_1$ :

$$H_0 : \beta_1 = 0, \text{ vs. } H_1 : \beta_1 \neq 0,$$

in the context of the Linear Regression model:

$$y_t = \beta_0 + \beta_1 x_t + u_t,$$

and the Durbin Watson (D-W) test based on the hypotheses:

$$H_0 : \rho = 0, \text{ vs. } H_1 : \rho \neq 0,$$

in the context of the Autocorrelation-augmented Linear Regression (AALR) model:

$$y_t = \beta_0 + \beta_1 x_t + u_t,\ u_t = \rho u_{t-1} + \varepsilon_t.$$

(e) Using your answer in (d) explain why adopting the alternative in the case where $H_0$ was rejected by the D-W will be a bad idea.

**Problem 3 (60 minutes)**

Consider the Solow model with Cobb-Douglas technology and with population growth of $n$.

The model is:

$$Y(t) = F(K(t),\ L(t),\ t) = AK_t^{\alpha}L_t^{1-\alpha},\ 0 < \alpha < 1.$$
$$\dot{K} = I - \delta K = sY - \delta K.$$
$$L(t) = e^{tn}.$$

Given this model, answer the following.

(a) Show that $\dot{k} = sAk^{\alpha} - (n+\delta)k,\ k \equiv K/L.$

(b) Find the steady state values for $k$ and for $y \equiv Y/L.$

(c) Assume the initial per capita capital stock is below the steady state. Let $\gamma_k \equiv \dot{k}/k$ and show that $\partial\gamma_k/\partial t < 0$ and $\partial^2\gamma_k/\partial t^2 > 0.$ What happens to the growth rate $\gamma_k \equiv \dot{k}/k$ as $t \rightarrow \infty$ ? Explain.

(d) Suppose that initially at time $t = 0$ the economy is in its steady state and there is an increase in the population growth rate from $n_0$ to $n_1 > n_0.$ Graph the time path of $\ln k$ and $\ln K$ from time 0 on. Be sure to label and explain each graph thoroughly.

(e) Assume capital and labor are paid their marginal products. Describe the time path of the real wage and the marginal product of capital as the economy converges to its new steady state.

**Problem 4 (30 minutes)**

At time 1 in the country of Freedonia, President Rufus T. Firefly announces (unexpectedly) that beginning at time 10 money growth will decrease from its current and past long run value of 6% to 2%. Given this change, determine the time path, past, current and future, of

(a) the nominal interest rate;

(b) the real demand for money;

(c) the income velocity of money;

(d) inflation.

Be sure to explain each of your answers fully.