Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam

Summer 2007

PART I

June 25, 2007

Please answer all 4 questions. Notice the time allotted to each question.

Problem # 1 (30 minutes)

John resides on an island separated from the rest of the world. There are two products that he can "produce", gathering food, F, and making making cloths, C. The production function for food is $F = \sqrt{L_F}$ and that for cloths is $C = \sqrt{L_C}$, where L_F and L_C are the hours he spends each months on producing food and cloths, respectively. John's utility function is $U(F,C) = \sqrt{F \cdot C}$ and he has decided to spend each month 250 hours in labor.

(a) How will John choose to allocate his labor each month? What will be the quantities of food and cloths that he will produce? What will be his utility?

(b) Suppose the world discover John on the island and are willing to trade with him food and cloths at a price ratio of $P_F/P_C = 1/2$, but are unwilling to help him off the island. If John continues to produce each month the quantities of food and cloths you have derived in part (a), what will he choose to consume, given the opportunity to trade? What will his utility be?

(c) How would your answer to (b) change if John adjusts his production to take advantage of the world's prices?

Problem # 2, (30 minutes)

Consider a market consisting of two firms that produce similar products. Let q_i be the quantity produced by firm i, $i = 1, 2$. Both firms use the same inputs, x and y. The production functions are:

$$q_1 = \min\{2x - y, 2y - x\}$$

$$q_2 = 0.25 \cdot \max\{2y - x, 2x - y\}.$$

For the production function of q_1 you may disregard combinations of the inputs which result in negative quantities of the output.

The inputs can be bought in competitive markets at prices p_x and p_y. The market demand functions for the two goods are

$$q_1 = 100 - 2p_1 + p_2$$

$$q_2 = 80 + 2p_1 - p_2$$

where p_1 and p_2 are the market prices of products 1 and 2, respectively. Also, neither firm has fixed cost.

(a) What is the total cost function of each firm?

Suppose now that $p_x = 3$ and $p_y = 4$.

(b) If the two firms compete, what price will each charge in equilibrium?

(c) Suppose firm 2 offers the shareholders of firm 1 to purchase their shares and merge. Furthermore, once the two firms merge the present shareholders of firm 1 will be guaranteed 40% of the profits of the merged firm. Should the shareholders of firm 1 accept the merger under these condition? Briefly explain your answer.

Problem # 3, (60 minutes)

Consider an exchange economy with two consumers and two goods. The consumption sets are $X^1 = X^2 = \mathbf{R}^2_+$. The initial endowments are $\omega^1 = (8,0)$ and $\omega^2 = (0,2)$. The preferences of the first consumer are represented by the utility functions $u^1(x_1,x_2) = \min\{x_1,x_2\}$, and the second consumer's preferences are represented by the utility function $u^2(x_1,x_2) = x_1x_2$.

(a) Derive the competitive equilibrium (quantities consumed and prices) of this economy. What are the utility levels of the two consumers in that equilibrium?

Consider now the following situation. Before trade takes place each consumer can destroy part of his initial endowment. After the destruction takes place the two consumers have the **reduced endowments**

$$e_1 = (\alpha,0) \text{ with } 3 \leq \alpha \leq 8, \text{ and } e_2 = (0,\beta) \text{ with } 0 < \beta \leq 2,$$

and enter the market with these endowments. The pure exchange economy

$$E(\alpha,\beta) = (X^i,u^i,e^i)_{i=1,2}$$

has a unique competitive equilibrium allocation, with equilibrium consumption bundles $x^i(\alpha,\beta)$ for consumers $i = 1,2$. The situation described above can be modeled as a two person game in strategic form

$$\Gamma = (I,(S_i)_{i\in I},(u_i)_{i\in I})$$

with $I = \{1,2\}$, $S_1 = [3,8]$, $S_2 = (0,2]$, and $u_i(\alpha,\beta) = v^i(x^i(\alpha,\beta))$ for $(\alpha,\beta) \in S_1 \times S_2$, $i \in I$.

(b) What can be said about the equilibrium payoffs of this game? How are they compared to the equilibrium utilities in part (a)? What can be said about the equilibrium values of α and β ?

Problem # 4, (60 minutes)

Consider an oligopolistic market with two firms, A and B, that produce identical products. Total market demand is given by

$$P(Q_A + Q_B) = 30 - (Q_A + Q_B)$$

where Q_A and Q_B are the quantities produced by the two firms, respectively. Both firms have no variable cost but each has fixed entry cost given by $C_A > 0$ and $C_B > 0$, where C_i is the fixed entry cost of firm i.

(a) Consider the two-stage von Stackelberg situation. Suppose firm A first decides if to enter or not and, if it enters the market, acts as the von Stackelberg leader. Subsequently, firm B decides if to enter the market or not and if it enters acts as a von Stackelberg follower. Formulate this situation as an extensive form game. Be careful in your formulation. Formulate clearly the histories in this game and the exact payoff functions.

(b) For the game formulated in (a), consider the case $C_A < C_B$. Derive all combinations of the fixed cost (C_A, C_B) for which at least one subgame perfect Nash equilibrium of the game results in only one firm entering this market.

(c) Same as (b) when $C_A > C_B$.

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Summer 2007

PART II

June 28, 2007

Please answer all 5 questions. Notice the time allotted to each question.

Problem # 1, (30 minutes)

The class of estimators considered by the Gauss-Markov Theorem for the multiple regression model, $Y = X\beta + \varepsilon,$ can be categorized as $\widetilde{\beta} = AY$ (linear) and $AX = I$.

(a) Assuming that A is fixed, show that $\widetilde{\beta}$ is unbiased if and only if $AX = I$ and $E(\varepsilon) = 0$.

(b) Use this result to show that, so long as W is fixed and $(W^T X)^{-1}$ is well defined, any estimator of the form

$$\widetilde{\beta} = (W^T X)^{-1} W^T Y$$

is unbiased.

Problem # 2, (30 minutes)

A typical traditional econometric textbook syllogism concerning ’error Autocorrelation correction’ goes as follows.

(a) I estimated a theoretical relationship ($y = \beta_0 + \beta_1 x$) using OLS yielding:

$$\widehat{\beta} = (X^T X)^{-1} X^T y \tag{1}$$

where $\beta := (\beta_0, \beta_1)$.
The empirical results seem *reasonable* (signs, magnitudes, *t*ratios, R^2, etc.), but the Durbin-Watson (D-W) statistic seems low (close to zero), indicating that my model suffers from **error autocorrelation.**

(b) I know that the OLS estimator $\widehat{\beta}$ is still **unbiased** and **consistent**, but no longer **efficient**; the GLS estimator

$$\widetilde{\beta} = (X^T \Omega^{-1} X)^{-1} X^T \Omega^{-1} y \tag{2}$$

where $\Omega := Cov(u \mid X)$, is more efficient than $\widehat{\beta}$ in (1).

(c) I also know that I can still use $\widehat{\beta}$ to perform valid inference concerning (β_0, β_1) using **autocorrelation-consistent standard errors.**

(d) A classic cure of the autocorrelation problem is provided by the feasible GLS

$$\widetilde{\beta}^* = (X^T \widehat{\Omega} X)^{-1} X^T \widehat{\Omega}^{-1} y \tag{3}$$

based on the Autoccrrelation-Corrected Linear Regression (ACLR) model

$$y_t = \beta_0 + \beta_1 x_t + u_t, \qquad u_t = \rho u_{t-1} + \varepsilon_t \tag{4}$$

(3)provides an ’optimal’ estimator for (β_0, β_1) and ensures the reliability of inference.

Explain the claims (a)-(d) and discuss critically these arguments pointing out for each claim what follows (and under what circumstances) and what doesn’t.

Problem # 3, (30 minutes)

Prior to 1997, Churchill County, Nevada had no history of leukemia (a form of cancer). Between 1997 and 2001, 16 children were diagnosed with leukemia. In a recent AER paper, Lucas Davis investigated the impact of this "cancer cluster" on housing prices. He collected data on the timing of leukemia cases and on the prices and characteristics of 2,495 homes sold in Churchill County between 1990 and 2002. He collected similar data on 3,683 homes in a neighboring county (Lyon) that did not report any cases of leukemia. What follows is a simplified description of his econometric analysis.

(a) First, Davis pooled the data across all homes (j=1,...,6178) and months $(t = Jan/1990,\ldots,Dec/2002)$. Then he estimated the following equation by OLS:

$$PRICE_{jt} = \beta X_{jt} + \lambda RISK_{jt} + \mu_{jt}, \tag{1}$$

where $PRICE_{jt}$ is the sale price of house j in month t, X_{jt}*is* a vector of characteristics describing that home (e.g. lot size, square feet, age) and $RISK_{jt}$ is a proxy for the perceived risk of developing leukemia, as measured by the comulative number of leukemia cases diagnosed since 1990 in the county where house j is located.

The unexplained portion of $PRICE_{jt}$ can be decomposed into time-constant component and a time-varying component: $\mu_{jt} = a_j + \varepsilon_{jt}$. If $E[\varepsilon_{jt} \mid X, RISK] = 0$ for all j and t, what must be assumed about a_j for the OLS estimator of λ in equation (1) to be unbiased?

(b) Many of the homes in Churchill and Lyon were sold multiple times between 1990 and 2002. Davis used these repeat sales to derive a second estimating equation. First, he subtracted the average sale price of house j from both sides of (1) to yield the following equation:

$$PRICE_{jt} - \overline{PRICE}_j = \beta X_{jt} + \lambda RISK_{jt} + \mu_{jt} - \overline{PRICE}_j, \tag{2}$$

where $\overline{PRICE}_j = T_j^{-1} \sum_t PRICE_{jt}$ for house j that was sold T_j times between 1990 and 2002. Rewrite the right-hand side of (2) as a as a function of $\beta,\ \lambda,\ X_{jt},\ \overline{X},\ RISK_{jt},\ \overline{RISK}_j,\ \varepsilon_{jt}$, and $\overline{\varepsilon}_j$. (This is Davis's second estimating equation.

(c) Davis finds that the model you derived in part (b) generates an estimate for λ that is nearly identical to the estimate for λ from equation (1). Their 95% confidence intervals overlap. What do these results suggest about the data generating process for $RISK_{jt}$ and μ_{jt} ?

Problem # 4, (60 minutes)

Consider the Ramsey model with (1) no technological progress, (2) no population growth, (3) identical agents where (4) each agent offers one unit of labor at each time t.

Each agent in the economy has capital k_t at time t and the rents capital to firms at the rental rate r_t. In addition, each agent offers a unit of labor at each time t and receives a real wage of w_t. The economy consists of an equal number of agents and firms. The output of each firm at time t is given by $y_t = Ak_t^\alpha l_t^{1-\alpha} = Ak_t^\alpha$ since the work effort $l_t = 1$, and $0 < \alpha < 1$. The instantaneous utility function of each agent is $u(c) = \ln(c)$.

Each individual solves the following problem

$$\text{maximize} \qquad U = \int_0^\infty e^{-\rho t} u(c_t) dt$$

$$\text{s.t.} \qquad dk/dt = r_t k_t + w_t - c_t - g_t - \delta k_t$$

in which k is the capital stock, r is the interest rate, c is consumption, δ is the rate of depreciation and g is government purchases (financed with lump sum taxes). The government maintains a balanced budget at all times. Assume for now that $g_t = \bar{g}$, a constant.

(a) Write the individual's lifetime budget constraint. That is, what must the present value of lifetime consumption equal?

(b) Write the Hamiltonian, the Euler equations and the transversality condition for the individual's optimization problem.

(c) Determine the consumption of an individual at time t as a function of his wealth at time t, the present value of his lifetime labor income and the present value of his lifetime taxes.

(d) In equilibrium, $r_t = \alpha A k_t^{\alpha-1}$ and $w_t = (1-\alpha)k_t^\alpha l_t^{-\alpha} = (1-\alpha)k_t^\alpha$. Given these conditions, determine the steady state values for the capital stock, consumption, r_t and w_t.

(e) Construct the phase diagram for c and k, showing the dynamic path towards the steady state equilibrium.

(f) Suppose that at time 0 the economy is in its steady state equilibrium and there is a **permanent** increase in government purchases g . Describe the effects that this will have on the steady state values of k, c, w and r. Describe the effects this will have on the loci of $dc/dt = 0$ and $dk/dt = 0$, and determine the time path to the new steady state.

(g) Consider a **temporary** increase in government expenditures g. Suppose for example that at time 0 the economy is in its steady state and suppose that the new path for government purchases is

$$g_t = \bar{g} + \triangle g \text{ for } t \leq s$$
$$g_t = \bar{g} \text{ for } t > s$$

for some $s > 0$ and $\triangle g > 0$. Determine the effects this will have on the steady state values of

$k,\ c$ and $r.$ Determine the effects of this change on the loci of $dc/dt = 0$ and $dk/dt = 0.$ What happens to consumption at time zero? Explain. Determine the saddle path taken to the steady state.

Problem # 5, (30 minutes)

In the country of Fredonia there is an unusually bad harvest season. Use the market clearing model of Baro to determine the effects this will have on the equilibrium values of output, Y, work effort, L, consumption, C, investment, I, the rate of interest, r, and the price level, P.