Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam

Spring 2010

PART I

May 31, 2010

Please answer all 4 questions. Notice the time allotted to each question.

Problem 1. (30 minutes)

Consider a case of an industry in which three firms that produce an identical product determine sequentially their level of output. Firms 1, 2 and 3 move in this order, each observing before it moves the quantity already chosen by each of its predecessors. Let q_i denote the quantity produced by firm i, $i = 1, 2, 3$. The market inverse demand function is given by $p = 2 - q_1 - q_2 - q_3$, and the average cost of each firm is $AC_i(q_i) = 1$, $i = 1, 2, 3$.

(a) Find the subgame perfect equilibrium of this game.

(b) Compare the equilibrium profit of each firm with that of the three-firm Cournot market.

(c) Describe a Nash equilibrium of this game that is not subgame perfect.

Problem 2. (75 minutes)

PART 1:

The economy of the island of Atlantix is a pure exchange economy with $L = 2$ goods labeled $k = 1, 2$ and $N = 2n$ consumers labeled $i = 1, \ldots, N$. All consumers have the same consumption set $\mathbf{R}^2_+$ and the same Cobb-Douglas preferences represented by the utility function

$$u_i(x_i^1, x_i^2) = x_i^1 x_i^2$$

for every consumer i and consumption bundles $x_i = (x_i^1, x_i^2) \in \mathbf{R}^2_+$.

There are two types of consumers:

The n consumers $i = 1, \ldots, n$ have the endowment bundle $\omega_i = (\omega_i^1, \omega_i^2) = (20, 20)$.

The n consumers $i = n + 1, \ldots, 2n$ have the endowment bundle $\omega_i = (\omega_i^1, \omega_i^2) = (20, 0)$.

(a) Determine the competitive equilibrium allocation $(x_i^*)_{i=1,\ldots,2n}$ for this economy.

PART 2:

The government wants to convert Atlantix into a wildlife reserve. It proposes to re-settle the inhabitants of Atlantix on the hitherto-uninhabited island Pacifix. On Pacifix, the settlers will enjoy the same two goods, will have the same preferences, and each will have the endowment bundle $\widetilde{\omega}_i = (\widetilde{\omega}_i^1, \widetilde{\omega}_i^2) = (21, 21)$. Since each will have a larger endowment than before, the government proposal seems attractive.

Suppose all consumers have moved from Atlantix to Pacifix. Then some will find that, indeed, they are better off than before, while others will be worse off and disappointed. In order to prove this claim, do the following:

(b) Determine the competitive equilibrium allocation $(x_i^{**})_{i=1,\ldots,2n}$ for the new Pacifix economy.

(c) Compare the equilibrium utilities $u_i(x_i^*)$ and $u_i(x_i^{**})$ for $i = 1, ..., 2n$.

(Problem continues on next page).

PART 3:

In parts 1 and 2, not everybody was better off in the Pacifix economy, even though everybody had a larger endowment. But some of the consumers were better off. The latter holds true more generally: If, *ceteris paribus*, one economy is endowed with more resources, then some of its consumers have higher equilibrium utility than their counter-parts in the economy with less resources. Your task is to show this under the following assumptions.

Let the number of commodities, L, and the number of consumers, N, be given. Moreover, let each consumer $i = 1, \ldots, N$ have consumption set $X_i = \mathbf{R}^L_+$ and monotone preferences represented by a given utility function u_i. We consider two pure exchange economies, $\mathcal{E} = (u_i, \omega_i)_{i=1}^N$ and $\mathcal{E}' = (u_i, \omega_i')_{i=1}^N$. Notice that the consumers in $\mathcal{E}$ and $\mathcal{E}'$ have the same preferences. We assume:

(A) $\sum_{i=1}^N \omega_i' \gg \sum_{i=1}^N \omega_i$,

that is, the aggregate endowment of economy $\mathcal{E}'$ is strictly larger than the aggregate endowment of economy $\mathcal{E}$. Let (p^*, x^*) be a competitive equilibrium of $\mathcal{E}$ and let (p^{**}, x^{**}) be a competitive equilibrium of $\mathcal{E}'$. Consider the allocation $x' = (x_1', \ldots, x_N')$ given by

$$x_i' = x_i^* + \frac{1}{N}\left[\sum_{j=1}^N \omega_j' - \sum_{j=1}^N \omega_j\right]$$

for $i = 1, \ldots, N$.

(d) Show that x' is a feasible allocation of the economy $\mathcal{E}'$.

(e) Show that $u_i(x_i') > u_i(x_i^*)$ for all i.

(f) Show that $u_i(x_i^{**}) > u_i(x_i^*)$ for at least one i.

Hint: Use (e) above to show that if assertion (e) does not hold then the conclusion of the first welfare theorem with regard to x^{**} fails to hold.

Problem 3. (45 minutes)

Consider two firms that produce (each in its own plant) an identical product, q, and use the same inputs, x and y. The product is sold in a perfectly competitive market at a price p_q and the two inputs are purchased in perfectly competitive markets at prices p_x and p_y respectively. The production functions of the plants each firm uses are

$$q_1 = \text{Min}\{x_1^{1/2}, y_1^{1/2}\}$$
and
$$q_2 = \frac{1}{2} \cdot \text{Max}\{x_2^{1/2}, y_2^{1/2}\}.$$

Also, neither firm has fixed cost.

(a) Derive the variable cost function of each firm.

(b) Derive the (non contitional) demand functions of each firm for each of the two inputs. (Altogether you should derive four demand functions).

(c) Suppose the two firms merge. Derive the (non conditional) demand functions of the merged firm for each of the inputs. (Hint: Notice that the merged firm can use either or both of the plants it has at its disposal).

(d) Now suppose the government, wishing to reduce the use of input x, imposes a tax of $t > 0$ on each unit of x in excess of $\overline{x}$ used in each plant. How will this tax affect your answers to parts (b) and (c) above?

Problem 4. (30 minutes)

A crime is observed by n people. Each of them would like the police to be informed but each prefers that someone else make the phone call. Precisely, suppose that each person attaches the value v to the police being informed and bears the cost c if he calls the police himself, where $v > c > 0$. The situation is then modeled as a game in strategic form in which the strategy set of each person is $\{Call,\ Don't\ Call\}$.

(a) Derive all the pure-strategy Nash equilibria of this game.

In the following we are interested in the symmetric mixed-strategy Nash equilibria. Let p be the probability with which each person calls.

(b) Determine the value of p in a symmetric Nash equilibrium.

(c) Consider the events: "person 1 calls" and "no one calls". Given this symmetric Nash equilibrium, determine the respective probabilities of these two events. How do these probabilities change as n increases?

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PART II

June 3, 2010

Please answer all 4 questions. Notice the time allotted to each question.

Problem 1. (60 minutes)

1. Consider the data generating process

$$y = X\beta + \varepsilon \quad \varepsilon \sim ID(0, \Omega), \tag{(1)}$$

where Ω is diagonal and $E[\varepsilon|X] = 0$. Please answer the following questions:

(a) (a) Prove that the OLS estimator $\hat{\beta}_{OLS} = (X'X)^{-1}X'y$ is unbiased and derive its (conditional) sampling variance.

(b) Prove that the GLS estimator, $\hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y$ is unbiased and derive its sampling variance.

(c) Prove that the GLS estimator is more efficient that the OLS estimator.

(d) Cragg (1983) and White (1982) introduced the partial generalized least squares estimator. For $Z = [X \quad W]$, where W is an $n \times G$ matrix such that W has full column rank, define the partial GLS estimator as

$$\hat{\beta}_{PGLS} = \left(X'Z(Z'\Omega Z)^{-1}Z'X\right)^{-1} X'Z(Z'\Omega Z)^{-1}Z'y. \tag{(2)}$$

i. (d.1) Show that when $W'X = 0$, $\hat{\beta}_{PGLS} = \hat{\beta}_{OLS} - (X'X)^{-1}X'\Omega W(W'\Omega W)^{-1}W'y$. Recall that

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} V_1^{-1} & -V_1^{-1}BD^{-1} \\ -D^{-1}CV_1^{-1} & D^{-1} + D^{-1}CV_1^{-1}BD^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} A^{-1} + A^{-1}CV_2^{-1}BA^{-1} & -V_2^{-1}BA^{-1} \\ -A^{-1}CV_2^{-1} & V_2^{-1} \end{bmatrix}$$

where $V_1 = \left(A - BD^{-1}C\right)$ and $V_2 = \left(D - CA^{-1}B\right)$.

(d.2) Show that the sampling variance of the PGLS estimator is

$$V(\hat{\beta}_{PGLS}) = \left(X'Z\left(Z'\Omega Z\right)^{-1} Z'X\right)^{-1}. \tag{(3)}$$

(d.3) Show that $\hat{\beta}_{PGLS}$ is more efficient that $\hat{\beta}_{OLS}$, but less efficient than $\hat{\beta}_{GLS}$.

(Problem continues on next page.)

(d.4) Show that a necessary condition for W to increase the efficiency of the PGLS estimator is

$$(X'\Omega X)^{-1}X'\Omega W \neq (X'X)^{-1}X'W. \qquad ((4))$$

Interpret what this condition means in regression terms. That is, think of either X or $\Omega^{1/2}X$ as regressors and W or $\Omega^{1/2}W$ as regressands.

Problem 2. (30 minutes)

An analyst is examining the statistical adequacy of a bivariate linear regression model,

$$y_t = \beta_o + \beta_1 x_t + u_t$$

with $t \in [1, N]$, $x_1 ... x_N$ fixed in repeated samples, and (purportedly) $u_i \sim niid(0, \sigma^2)$. (Or, if you prefer, you can replace these assumptions with those of the LR model in the Probabilistic Reduction Specification.) In the course of this examination, the analyst estimates an auxiliary regression, in which the dependent variable is some function of the fitting errors from the original model, and performs hypothesis tests using the estimated auxiliary regression equation.

1. (a) In what fundamental way does a test on the estimated coefficients in the auxiliary regression equation differ from a hypothesis test on a coefficient, or combination of coefficients, in the original regression model?

   (b) What kind of auxiliary regression would be suitable for examining the adequacy of the assumption with regard to model serial dependence — or "model error non-autocorrelation" in the textbook regression formulation? What sorts of model re-specification might the results from this auxiliary regression suggest?

   (c) What kind of auxiliary regression would be suitable for examining the adequacy of the assumption with regard to homoskedasticity (in the textbook specification) or with respect to "t-inhomogeneity" in the Probabilistic Reduction specification? What sorts of model re-specification might the results from this auxiliary regression suggest?

   (d) What kind of auxiliary regression would be suitable for examining the adequacy of the linearity assumption in the statistical generating mechanism given above? What sorts of model re-specification might the results from this auxiliary regression suggest?

   (e) Suppose that this examination of the statistical adequacy of the original model formulation suggests a more complicated model specification, in which a particular explanatory variable is predicted to enter with a non-zero coefficient. The analyst estimates this model and observes that this variable enters with an estimated coefficient which is right on the borderline of being statistically significant, using the usual cultural norms for statistical significance. Has the analysis involved in this examination of the statistical adequacy of the original model altered the meaning/validity of this hypothesis test on the newly-included variable?

Problem 3. (30 minutes)

Denote:

$$\begin{aligned} w &: \text{log of wage rate.} \\ Age &: \text{age.} \\ Exp &: \text{working experience.} \\ Ed &: \text{years of schooling} \\ Gender &: \text{gender (gender} = 1 \text{ if male, and 0 otherwise).} \end{aligned}$$

The production function of human capital is sometimes estimated by regressing the log of the wage rate on education, age, experience and other relevant variables. For simplicity, suppose the following model is the true model that generates the wage data.

$$w_i = \alpha_0 + \alpha_1 Age_i + \alpha_2 Exp_i + \alpha_3 Ed_i + \alpha_4 Ed_i^2 + \alpha_5 Gender_i + \varepsilon_i$$

where ε_i is the error term.

(a) Suppose the model is estimated using the ordinary least squares (OLS). Under what assumptions is the OLS estimate the best linear unbiased estimate (BLUE)?

(b) What is the marginal effect of education on the log of the wage rate at the mean education level *Edmean*? Show and explain how you would obtain a 95% confidence interval on this estimate.

(c) What is the optimal education level (ED^*) for maximizing the log of wages from this model? Show and explain how you would test the one-sided hypothesis $H_0 : ED^* \geq 16$.

(d) What is the economic interpretation of the coefficient α_5 in the model?

(e) Suppose the experience variable is not observed for the given dataset. What is the consequence if the model is estimated without using this variable? Since individuals do not begin acquiring experience until they finish their education, a commonly used proxy for experience is

$$Exp^* = Age - Ed - 5.$$

Can we use Exp^* in place of Exp in our regression? Explain your answer.

(f) Suppose both wage rate and education are positively correlated to an unobservable variable: ability. People with higher ability tend to have more education and higher wage rate. What assumption of the usual model is violated? Describe a method to correct this violation.

Problem 4. (60 minutes)

Note: Explain your answers fully but succinctly. If graphical analysis is called for, explain all shifts in curves.

Consider the Freedonian economy in which output is a function of capital only and in which the marginal return to capital is constant. Specifically, assume that time t output (income) is

$$y_t = (1 + r)k_{t,} \tag{(1)}$$

where $k_t > 0$ is the time t capital stock and $r > 0$ is the constant rate of return to capital. The time t budget constraint for an individual is

$$c_t + k_{t+1} = (1 + r)k_t \tag{(2)}$$

where $c_t > 0$ is time t consumption. Given k_t one can use equation (2) for $t = 1, 2, ...,$ to show that the lifetime, time t budget constraint is

$$\sum_{t=1}^{\infty}[c_t/(1 + r)^{t-1}] = (1 + r)k_1. \tag{(3)}$$

Each individual wishes to maximize lifetime utility given by

$$\sum_{t=1}^{\infty}\beta^{t-1}\ln(c_t),\ \beta \equiv (1 + \theta)^{-1}$$

subject to the lifetime budget constraint, where $0 < \beta < 1$.

(a) Derive the first order conditions for this problem and show that in equilibrium $c_{t+1} = c_1[(1 + r)/(1 + \theta)]^t$.

(b) Given the budget constraint and the results in (a), determine time 1 consumption as a function of k_1.

(c) Derive k_2 as a function of k_1.

(d) Determine the economy's rate of growth.

At time 1 the Freedonian government decides to impose a permanent tax on capital income at a rate of $\tau_{cap} > 0$. Thus the *after -tax* income at time t is $\widehat{y}_t = (1 - \tau_{cap})(1 + r)k_t$. Assume that $(1 - \tau_{cap})(1 + r) > 1 + \theta$ and answer the following questions.

(e) Derive time 1 consumption as a function of k_1.

(f) Derive k_2 as a function of k_1.

(g) How does the imposition of the tax on capital affect the economy's growth rate (the one derived in part (d) above)?

(Problem continues on next page).

Now suppose that instead of a tax on capital income the government taxes consumption at a rate of $\tau_{con} > 0$. Consequently, the time t budget constraint is $(1 + \tau_{con})c_t + k_{t+1} = (1 + r)k_t$, and the new lifetime budget constraint is

$$\sum_{t=1}^{\infty}[(1 + \tau_{con})c_t/(1 + r)^{t-1}] = (1 + r)k_1. \qquad ((4))$$

(h) Derive the first order conditions for this problem.

(i) Derive time 1 consumption as a function of k_1.

(j) Derive k_2 as a function of k_1.

(k) How does the imposition of the tax on consumption affect the economy's growth rate (the one derived in part (d) above)?

(l) How does the switch from a tax on capital to a tax on consumption affect the economy's growth rate?