Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam

March 2012

PART I

March 12, 2012

Please answer all 4 questions. Notice the time allotted to each question.

Problem 1. (45 minutes)

A consumer consumes three goods, 1 and 2 and 3. His preferences over consumption bundles $(x_1, x_2, x_3) \in \mathbf{R}^3_+$ can be represented by the utility function

$$u(x_1, x_2, x_3) = \min\{x_1,\ x_2^{1/2} \cdot x_3^{1/2}\}$$

Let $p_i > 0$ be the price of good $i,\ i = 1, 2, 3$ and let $m > 0$ be this consumer's income.

(a) Derive this consumer's Marshallian demand for each good. (Hint: for any given prices and income what can one say about the properties of the chosen consumption bundle?).

(b) Derive his indirect utility function and his expenditure function.

(c) Derive the Hicksian (compensated) demand for each good.

(d) What portion of his income does this consumer spend on each good?

Problem 2. (45 minutes)

Consider a Cournot duopoly with inverse demand function $P(Q) = \begin{cases} \alpha - Q & \text{if } Q \leq \alpha \\ 0 & \text{if } Q > \alpha, \end{cases}$

where $\alpha > 0$ and Q is the aggregate output. The unit cost c_i of firm $i \in \{1, 2\}$ is random and can take any of the two values c_H, c_L (with $\alpha > c_H > c_L$). The type set of each firm is thus $T_1 = T_2 = \{c_H, c_L\}$, and a type profile is of the form $(c_1, c_2) \in T_1 \times T_2$. Assume that the joint probability distribution of the firms' types, μ, is given by

$$\begin{cases} \mu(c_H, c_H) = \mu(c_L, c_L) = 1/3; \\ \mu(c_L, c_H) = \mu(c_H, c_L) = 1/6. \end{cases}$$

Each firm observes the realized value of its own cost, but not that of the other firm. Nevertheless, the values of c_H and c_L, as well as the probability distribution μ, are known to both firms.

(a) Compute the conditional probabilities $\Pr(c_{-i} = c_H | c_i)$ and $\Pr(c_{-i} = c_L | c_i)$, for any $i \in \{1, 2\}$ and $c_i \in \{c_H, c_L\}$. Are the random variables c_1 and c_2 independent? Justify your answer.

(b) Derive the Bayesian Nash equilibrium of this model.

Problem 3. (45 minutes)

Part A:

Consider first a pure exchange economy with two commodities labled $k = 1, 2$ and $n \geq 2$ consumers labled $i = 1, \ldots, n$ (In (c) and (d) below production will be added). A consumption bundle for consumer i assumes the form $x_i = (x_i^1, x_i^2) \in \mathbf{R}_+^2.$ Consumer i has the initial endowment bundle $\omega_i = (1, \omega_i^2)$ with $\omega_i^2 > 0.$ Each consumer has preferences represented by the utility function

$$u_i(x_i^1, x_i^2) = x_i^1 - x_i^2 \text{ for } (x_i^1, x_i^2) \in \mathbf{R}_+^2.$$

(a) What is the difference between a competitive equilibrium with exact market clearing and a competitive equilibrium with frww disposal?

(b) What kind of competitive equilibrium do you obtain in the above pure exchange economy? Determine the equilibrium prices and quantities. (Consider price systems of the form $p = (1, p^{2)})$.

Part B:

Suppose now that in this economy there exists also a firm with technology

$$Y = \{y = (y^1, y^2) \in \mathbf{R}^2 \text{ such that } y^1 \leq 0,\ y^2 \leq a \cdot |y^1|\}$$

where $a > 0.$ Consumer i owns a share $\theta_i \geq 0$ of the firm, with $\sum_i \theta_i = 1.$

(c.1) Sketch the technology.

(c.2) What are the equilibrium prices and quantities in this production economy?

(c.3) How is the type of competitive equilibrium affected by the introduction of the technology?

(d) Suppose instead that the firm's technology is

$$Y = \{y = (y^1, y^2) \in \mathbf{R}^2 \text{ such that } y^2 \leq 0,\ y^1 \leq b \cdot |y^2|\}$$

where $b > 0.$ Sketch this technology and derive the competitive equilibrium prices and quantities.

Problem 4. (45 minutes)

A profit maximizing firm produces a single output q using a single input x. Its production technology is described by the production function

$$q = f(x) = \begin{cases} n \text{ if } x \in \mathbf{R}_+ \text{ and } n^2 \leq x < (n+1)^2,\ n = 0,1,2,3,4 \\ \\ 5 \text{ if x} \geq 25 \end{cases}$$

The firm sells its output at price $p > 0$ and purchases the input at a price $r > 0$.

(a) Draw a diagram of this production technology.

(b) Is this technology (i) closed, (ii) convex, (iii) additive? Briefly explain.

(c) Does this technology exhibit returns to scale, and if yes then which? Briefly explain.

(d) Suppose the firm sells its output in a perfectly competitive market. Derive its supply function $q(p)$ and its profit function $\pi(q)$.

(e) Now suppose the firm is a monopoly facing the inverse demand curve $p = 6 - 2q$. What quantity should it produce?

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PART II

January 15, 2012

Please notice:

Problems 1-3 are Econometrics problems
Problems 4-5 are Macro problems

Notice the time allotted to each problem.

Problem 1. (35 minutes)

Consider a linear regression model that has all the properties of the CLRM, except it exhibits group-wise heteroskedasticity.

Specifically, there are $g = 1 \ldots G$ groups in the data (think of something like "States" or "Counties"). Each group has dependent vector $\mathbf{y}_g$, data matrix $\mathbf{X}_g$, and error vector ϵ_g. Let n_g be the number of observations in each group, and $n = \sum_{g=1}^{G} n_g = n$ be the total sample size. Groupwise heteroskedasticity arises since $V(\epsilon_g) = \sigma_g^2 \neq V(\epsilon_h) = \sigma_h^2, \forall g \neq h$. However, the group-specific error terms are uncorrelated within and across groups.

Stacking the group-specific elements yields the full-sample regression model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \epsilon$$

1. Denote the full-sample variance-covariance matrix for ϵ as $\boldsymbol{\Omega}$. Show its explicit form in terms of $\sigma_g^2, g = 1 \ldots G$.
2. You first decide to estimate each group separately via OLS. For generic group g, show the matrix form of the OLS solution and its variance. Call the OLS solution $\mathbf{b}_g$, and the variance $\mathbf{V}_g$.
3. Show the matrix form of the full-sample GLS solution $\hat{\boldsymbol{\beta}}$, and its variance $\mathbf{V}$ in terms of $\mathbf{X}$, $\mathbf{y}$, and $\boldsymbol{\Omega}$.
4. Now express $\hat{\boldsymbol{\beta}}$ in terms of sums over $\mathbf{X}_g$, $\mathbf{y}_g$, and σ_g^2. (*Hint*: Partition $\mathbf{X}$ and $\mathbf{y}$ into the G groups, and remember the rule for taking the transpose of a partitioned matrix.)
5. Now re-visit your group-wise OLS solution and note that $\mathbf{X}_g'\mathbf{y}_g = \mathbf{X}_g'\mathbf{X}_g\mathbf{b}_g$. Use this in combination with your result from the preceding question to express the solution for $\hat{\boldsymbol{\beta}}$ as a weighted sum of group-wise OLS solutions. How is the weight a given $\mathbf{b}_g$ receives related to its group-specific OLS variance $\mathbf{V}_g$?
6. Describe how you could derive a 2-step FGLS estimator for $\boldsymbol{\beta}$

Problem 2. (25 minutes)

You want to examine the determinants of beach visitation at the Outer Banks, NC. You have surveyed 227 households that visited this area for a full day in the summer of 2010. The survey elicited the number of minutes spent by the typical household member on a beach within the following 4-hour intervals: morning (8am-12 noon), afternoon (12-4pm), evening (4-8pm). Thus, there are three observations for each household, one per time segment, leading to a total sample size of 681.

For now assume the CLRM assumptions hold. You run a basic OLS regression of minutes (number of minutes spent on a beach in a 4-hour interval by a typical household member) on a constant term, "kids" (number of children in the household), "inc000" (annual household income in \$1000s), "firstvis" (1=first visit to the Outer Banks, 0=repeat visit), and "avgtemp" (average 4-hour interval air temperature, in degrees Fahrenheit).

The STATA results are as follows:

```
Number of obs =     681
F(  4,   676) =   23.83
Prob > F      =  0.0000
R-squared     =  0.1236
Adj R-squared =  0.1184
Root MSE      =  182.27

------------------------------------------------------------------------------
     minutes |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        kids |   19.10343   5.702982     3.35   0.001     7.905743    30.30112
      inc000 |  -.1621121   .1096565    -1.48   0.140    -.3774203    .0531961
    firstvis |  -21.71681   19.64906    -1.11   0.269    -60.29733    16.86371
     avgtemp |   16.67931   1.870286     8.92   0.000     13.00704    20.35158
       _cons |  -1107.108   139.7995    -7.92   0.000    -1381.602   -832.6149
------------------------------------------------------------------------------
```

Estimated variance-covariance matrix for **b** (only lower half shown)

```
                 kids       inc000     firstvis      avgtemp        _cons
    kids       32.524
  inc000  -.05040208   .01202454
firstvis   8.7858714  -.01199221   386.08548
 avgtemp    .1444278   .00736123   1.5125216    3.4979708
   _cons  -39.504998  -1.8583582   -177.2895   -259.87459    19543.899
```

Note that the constant is the last estimate reported in the regression output. Thus, for this problem, label your coefficients as follows: β_1=`kids`, β_2=`inc000`, β_3=`firstvis`, β_4=`avgtemp`, and β_5=`constant`.

1. Interpret the marginal effects of `kids` and `avgtemp` .

2. Which hypothesis is implicitly tested by the reported F-statistic? State the null hypothesis

(H_0) in mathematical terms involving all relevant parameters. Then write your H_0 as $\mathbf{R}\boldsymbol{\beta} = \mathbf{q}$, with exact specification of $\mathbf{R}$ and $\mathbf{q}$. What is your conclusion regarding your H_0 based on the reported F statistic and p-value ("Prob > ")? You can assume a 5% level of significance. .

3. Test the H_0 that the effect of an additional child in the household on beach time is identical to the effect of an additional degree F, ceteris paribus. State the null hypothesis in mathematical terms involving all relevant parameters. Then write your H_0 as $\mathbf{R}\boldsymbol{\beta} = \mathbf{q}$, with exact specification of $\mathbf{R}$ and $\mathbf{q}$. Numerically derive the F-statistic for this test using the output provided. State your test decision given a 5% critical F-value of 3.84. *(Use all provided decimals for the inputs into your computation. Report interim and final results to a precision of 4 digits.)*

4. Since there are three observations per individual, the data have a panel structure. Assume that there is a time-invariant household-specific error component (or "heterogeneity effect"). What would be the consequence of ignoring it and running OLS if (i) the unobserved heterogeneity effect is correlated with some regressors, (ii) the unobserved heterogeneity effect is NOT correlated with some regressors?

5. Assuming that the heterogeneity effect is correlated with some regressors. What would be the benefits and drawbacks of running a fixed effects (FE) model?

Problem 3. (30 minutes)

Consider the following special one-parameter case of the gamma distribution,

$$f(y) = \frac{y}{\lambda^2} exp\left(-\frac{y}{\lambda}\right), \quad y > 0, \lambda > 0$$

It can be shown that $E(y) = 2\lambda$ and $V(y) = 2\lambda^2$.

Suppose that in the true model, the parameter λ depends on regressors according to $\lambda_i = \frac{1}{2} exp(\mathbf{x}_i'\boldsymbol{\beta})$. Therefore, $E(y_i|\mathbf{x}_i) = exp(\mathbf{x}_i'\boldsymbol{\beta})$ and $V(y_i|\mathbf{x}_i) = \frac{1}{2} exp(\mathbf{x}_i'\boldsymbol{\beta})^2$.

Assume that the data are independent over i.

1. Derive the log-likelihood function (scaled by N^{-1}) for this gamma model.
2. Derive the score vector (i.e., the first order derivative of the LLF).
3. Show that the expressions for $E(y_i|\mathbf{x}_i)$ and $V(y_i|\mathbf{x}_i)$ imply that
   $E\left\{\mathbf{x}_i\left[(\mathbf{y}_i - exp(\mathbf{x}_i'\boldsymbol{\beta}))^2 - \frac{1}{2}(exp(\mathbf{x}_i'\boldsymbol{\beta}))^2\right]\right\} = 0$
4. Use the moment condition in part (3) to form a method-of-moment estimator $\hat{\boldsymbol{\beta}}_{GMM}$. State, in general terms, the optimal choice for the weight matrix W_N.

Problem 4. (45 minutes)

Consider an overlapping generation model in which individuals live for two periods. Utility is a function of consumption over two periods and is

$$U(c^y, c^o) = \ln c^y + \ln c^o,$$

in which c^y and c^o are consumption in the first period of life (when young) and in the second (when old), respectively. The time t young provide 1 unit of labor and earn the competitive real wage of w_t. The young must save in order to consume in old age.

L_t denotes the population size of the time t young generation. Assume $L_{t+1} = 1.03 \cdot L_t$, so generational cohort is growing at a fixed rate of 3%.

Time t output is $Y_t = K_t^{1/2} L_t^{1/2}$ where K_t is time t capital stock.

Answer the following, being sure to **explain** your results and being sure to show all of your calculations.

(a) Solve the time t young person's utility optimization problem; that is, derive optimal consumption and saving for the individual. The budget constraints are

For time t : $c_t^y = w_t - s_t$. For time $t+1$: $c_{t+1}^o = (1 + r_{t+1})s_t$, in which s_t is time t saving and r_{t+1} is the return to saving (equal, in equilibrium, to the time $t+1$ rental rate of capital).

(b) Given your answers in (a) determine the steady state value for k, the capital/labor ration.

(c) Is the competitive equilibrium dynamically inefficient?

Problem 5. (45 minutes)

This problem consists of three short-answer questions. All have equal weight.

Question 1:

Consider an endowment economy with a zero interest rate (goods can be stored at no cost, so the gross return to saving is 1). Agents live for three periods and there are overlapping generations. At any point in time there will be (1) the young, (2) the middle-aged and (3) the old. There is no population growth and each individual's output endowment is the 3-tuple (y^y, y^m, y^o), where superscripts indicating the stage in life. in the first stage of life individuals **cannot** borrow, so $c_t^y \leq y_t^y$. Utility is

$$U(c^y, c^m, c^o) = \ln c^y + \ln c^m + \ln c^o.$$

Answer the following, being sure to **explain** your results and being sure to show all of your calculations.

(a) Solve the individual's utility maximization problem. That is, determine the optimal desired consumption vector $(c^y, c^m c^o)$ by solving

$$\text{Max } U(c^y, c^m, c^o), \text{ subject to: } c^y + c^m + c^o = y^y + y^m + y^o \text{ , } c^y \leq y^y.$$

When solving this problem be sure to determine the conditions under which saving by a *young* agent is 0; that is, determine when $c^y = y^y$.

(b) Assume the endowment vector is $(y^y, y^m, y^o) = (1, 1/2, 0)$. What is the optimal consumption vector in this case? What is total saving rate at each point in time for the aggregate economy, i.e. what is total time t saving as a fraction of total output (total endowment)?

Question 2:

The Freedonian economy is populated by identical, infinitely lived individuals. Each individual is endowed with y units of non-storable output at each period of time. Lifetime utility is

$$U = \sum_{t=1}^{\infty} u(c_t)/(1+\theta)^{t-1}, \ u' > 0 \text{ and } u' < 0.$$

The Freedonian government can borrow from abroad at the fixed real interest rate of $r = \theta$. **This option is not available to individuals in Freedonia.** By time 1 the government has accumulated a real foreign debt of B units of output. This loan must be paid for with current and future lump-sum taxes. Taxes at time t are τ_t. From time 1 forward all taxes go to service the foreign debt.

Find the optimal vector of lump-sum taxes maximizing the lifetime utility of individuals in Freedonia. Be explicit. Specify and solve the problem, explaining the intuition behind your results.

(*Question 3 is on the following page)*

Question 3:

Transylvania has pegged its currency, called a "Lugosi", to the U.S. dollar. The targeted exchange rate is $\varepsilon = 5$ (Lugosi’s)/1 (U.S. dollar) = £5/$1.

Let P^{US} be the U.S. price level and P^{T} be the Transylvanian price level. That is, $P^{US} = \$/U.S.\ output$ and $P^{T} = Lugosi's/Transylvanian\ output$.

Purchasing power parity requires that at each time period $(\varepsilon_t P_t^{US})/P_t^{T} = 1.$ **Assume purchasing power parity holds. Assume also that the U.S. price level is constant.**

(a) At time 0 the Transylvania government has domestic debt of B_0 (in Lugosi’s) and a foreign debt of $\widetilde{B}_0$ (in U.S. dollars). What is the time 0 value of total debt measured in Lugosi’s? What is the real value of the total debt in units of Transylvanian output?

(b) Before time 0 things have worked well in Transylvania. However, at time 0, doubt begins to grow as to future government fiscal surplus. In fact, at time 0 there is a general consensus that the present value of future Transylvanian fiscal surpluses has decreased. How will this affect the Transylvanian price level, the rate of inflation and the exchange rate? Under what conditions would Transylvania be forced to default on its foreign debt?