Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam January 2013

PART I

January 14, 2013

Please answer all 3 questions. Notice the time allotted to each question.

Problem 1 **(80 minutes)**

A pure exchange economy $\mathcal{E} = \{(X^i, \succsim^i)_{i=1}^I, \omega\}$ with two consumers $i \in \{1, 2\}$ and two commodities $l \in \{1, 2\}$ is considered. The initial endowment of consumer 1 is $\omega^1 = (0, 0)$ and that of 2 is $\omega^2 = (10, 10)$. Each consumer i chooses among commodity bundles in the set $X^i = \mathbb{R}^2_+$ according to his/her preferences $\succsim^i$. Preferences of consumer 1 have the lexicographic form, i.e. for any $x^1, y^1 \in \mathbb{R}^2_+$, $(x^1_1, x^1_2) \succsim^1_{lex} (y^1_1, y^1_2)$ if either $x^1_1 > y^1_1$ or $x^1_1 = y^1_1$ and $x^1_2 \geq y^1_2$. Preferences of consumer 2 are described by the Cobb-Douglas utility function $u(x^2_1, x^2_2) = (x^2_1)(x^2_2)$. Let $p \in \mathbb{R}^2$, $p \neq 0$, be a price vector.

(*a*) Show that consumer 1's preferences are not continuous.

(*b*) Draw the Edgworth box for the economy $\mathcal{E}$ and determine the set of Pareto optimal allocations.

(*c*) Which of the Pareto-optimal allocations can be supported *only* by a quasi-price equilibrium with transfers (QPET)? Determine the price vector p, the distribution of wealth levels m^1 and m^2, and transfers T^1 and T^2.

(*d*) Which of the Pareto-optimal allocations can be supported by a price equilibrium with transfers (PET)? Determine the price vector p, the distribution of wealth levels m^1 and m^2, and transfers T^1 and T^2.

(*e*) Suppose that $N \in \mathbb{N}$ consumers enter the economy $\mathcal{E}$. Each consumer has the same utility function and the same endowment as consumer 2. Is there a Pareto-optimal allocation in this economy which is supportable by Walrasian equilibrium. If "No", say why; if "Yes", then determine it.

Problem 2 **(50 minutes)**

Merry Lynch Industries (MLI) has a cost function

$$C(Q) = \frac{16}{3} \cdot Q^3 - 16Q^2 + 32Q.$$

It can

either operate in the perfectly competitive U.S. market where the prevailing market price is $\overline{P} = 25$

or become a monopolist on the island nation state of Miniscula where it would face a linear demand curve given by the inverse demand function $P(Q) = 28 - 8Q$ for $0 \leq Q \leq 3.5$.

(a) Derive the marginal cost function for MLI and show that $MC(Q) > 0$ for all $Q \geq 0$.

(b) What location is MLI going to choose?

(c) What is its maximum profit at its preferred location?

HINTS: A diagram might help you analyze the problem. But you still have to do the algebra. For your information, $\sqrt{576} = 24$ if needed.

Problem 3 (50 minutes)

This problem is about third degree price discrimination. The monopolist has a cost function $C(Q) = c \cdot Q$ with constant marginal cost $c > 0$. The monopolist faces demand from two groups, group 1 and group 2. The respective demand functions are

$D_1(P) = 4 - P$ (for $P \leq 4$) for group 1;

$D_2(P) = 2 - P$ (for $P \leq 2$) for group 2.

(a) Determine the aggregate demand function, $D(P) = D_1(P) + D_2(P)$.

(b) Draw the aggregate demand curve.

(c) Determine the aggregate inverse demand function.

(d) Determine the marginal revenue function resulting from (c).

(e) Draw MR and MC in a diagram.

(f) Determine the profit maximum, if the monopolist cannot price discriminate.

(g) Determine the profit maximum, if the monopolist can charge different prices to groups 1 and 2.

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Part 2

January 17, 2013

Answer all questions, taking note of the recommended time allocation for each question.

Problem 1 (30 minutes)

Answer the following questions about a Law of Large Numbers (LLN) and a Central Limit Theorem (CLT).

1. What is the difference between a Law of Large Numbers (LLN) and a Central Limit Theorem (CLT)? What roles do they play in the analysis of empirical data in economics?

2. Now specifically assume that $X_i \sim iid(\mu, \sigma^2)$ for $i = 1 \ldots N$, and let $\overline{X}$ stand for the sample mean of $X_1 \ldots X_N$.

   (a) What would a Law of Large Numbers (LLN) say about X_i in this case? What would it say about $\overline{X}$?

   (b) What would a Central Limit Theorem say about X_i in this case? What would it say about $\overline{X}$?

3. Which of the assumptions about the X_i listed below could be weakened and still allow for a CLT?

   (a) $\mathrm{Var}(X_i)$ are all the same.

   (b) X_i are independently distributed.

   (c) X_i all have the same population mean.

4. Suppose that Y_t is a time series which is a martingale difference sequence for $t = 1 \ldots T$, with $E(Y_t) = \mu$ and finite variance σ^2. If one further supposes that $E(|Y_t|^r)$ exists for some $r > 2$, then it can be shown that an LLN and a CLT still apply. Answering this question, however, only requires that you understand the meaning of the term 'martingale difference sequence'.

   (a) What does the martingale difference sequence assumption imply about the possibility of forecasting $E(Y_t)$ from a linear function of past values of Y_t? What does the martingale difference sequence assumption imply about the possibility of forecasting $E(Y_t)$ from a possibly nonlinear function of past values of Y_t? How would your answers differ if the Y_t were serially independent?

(b) What does the martingale difference sequence assumption imply about the possibility of forecasting $\text{var}(Y_t)$ from a linear function of past values of Y_t? What does the martingale difference sequence assumption imply about the possibility of forecasting $\text{var}(Y_t)$ from a possibly nonlinear function of past values of Y_t? How would your answers differ if the Y_t were serially independent?

Problem 2 (30 minutes)

Consider the following regression model

$$y_{it} = \alpha + x_{it}'\beta + u_{it}\,,\quad i = 1, \dots, N; t = 1, \dots, T$$

where α is a scalar and x_{it} is of dimension $K \times 1$. The error term has the structure of: $u_{it} = \mu_i + v_{it}$.

a) Assume that the true model satisfies the following conditions: $\mu_i \sim IIN(0, \sigma_\mu^2)$, $v_{it} \sim IIN(0, \sigma_v^2)$, and they are independent of each other and the x_{it}'s in the model. What are the statistical properties of the following estimators: OLS, Within Fixed Effect estimator (FE), First Difference estimator (FD), Random Effect (RE) GLS and RE Feasible GLS. State these properties without proofs.

b) How do the following estimators make use of the between-group variation: OLS, Within FE, Between Estimator, and RE GLS.

c) Now, consider a different scenario: the above model is based on a clustered cross-sectional data set (instead of a longitudinal data set). Then the *i* is the cluster index and *t* is the member within cluster. If this data set exhibits very low Intra-Class Correlation, would you expect the efficiency gain of RE compared to FE to be large or small? Why?

d) Consider a standard longitudinal data set again: let us write the above panel data model into a compact form: $y = Z\delta + u$, where *Z* is of dimension NT x (K+1) which includes the intercept and all the *X* variables. Please describe how you would test the poolability of the model across individual N (i.e., how would you determine to estimate the model as one single panel model or as separate N time-series models). Please state the null hypothesis, test statistics, the statistic distribution under the null and rejection rule. Please state clearly how you get each component of the test statistics.

Problem 3 (30 minutes)

Consider the following linear regression model:

$$\begin{aligned} &\mathbf{y} = \mathbf{X}\beta + \alpha\mathbf{r} + \epsilon, \quad \text{with} \\ &\epsilon \sim n\left(\mathbf{0}, \sigma^2\mathbf{I}\right), \\ &plim\left(\tfrac{1}{n}\mathbf{X}'\epsilon\right) = \mathbf{0}, \quad plim\left(\tfrac{1}{n}\mathbf{r}'\epsilon\right) \neq 0 \end{aligned} \tag{1}$$

Thus, regressor $\mathbf{r}$, with scalar coefficient α, is endogenous. You have a set of instruments available for $\mathbf{r}$, collected in matrix $\mathbf{Z}$, where $plim\left(\frac{1}{n}\mathbf{Z}'\epsilon\right) = \mathbf{0}$.

Part (a)
You decide to estimate the parameters in this model via 2SLS. Let $\tilde{\mathbf{X}} = \begin{bmatrix}\mathbf{X} & \mathbf{Z}\end{bmatrix}$, and express the first-stage regression as

$$\mathbf{r} = \tilde{\mathbf{X}}\gamma + \tau \tag{2}$$

where τ is a "well-behaved" error term, i.e. $plim\left(\frac{1}{n}\tilde{\mathbf{X}}'\tau\right) = \mathbf{0}$.

Express the fitted value of $\mathbf{r}$, call it $\hat{\mathbf{r}}$, in terms of $\tilde{\mathbf{X}}$ and $\mathbf{r}$. Denote the residuals from this first-stage regression as $\hat{\tau}$ and show that:

1. $\tilde{\mathbf{X}}'\hat{\tau} = \mathbf{0}$.
2. $\mathbf{X}'\hat{\tau} = \mathbf{0}$ and $\mathbf{Z}'\hat{\tau} = \mathbf{0}$
3. $\hat{\mathbf{r}}'\hat{\tau} = 0$
4. $plim\left(\frac{1}{n}\hat{\mathbf{r}}'\epsilon\right) = 0$

You can assume that $plim\left(\frac{1}{n}\mathbf{r}'\tilde{\mathbf{X}}\right)$ and $plim\left(\left(\frac{1}{n}\tilde{\mathbf{X}}'\tilde{\mathbf{X}}\right)^{-1}\right)$ are "well-behaved" finite-valued matrices.

Part (b)
Now consider the second stage of the 2SLS problem, where $\mathbf{y}$ is regressed against $\mathbf{X}$ and $\hat{\mathbf{r}}$, the fitted value of $\mathbf{r}$.

From the previous step, you know that $\mathbf{r} = \hat{\mathbf{r}} + \hat{\tau}$. Insert this expression for $\mathbf{r}$ in equation (1) from part (a) and show that the estimators for β and α generated by this second step are consistent.

(Hint: The second-stage model can be written as $\mathbf{y} = \begin{bmatrix}\mathbf{X} & \hat{\mathbf{r}}\end{bmatrix}\begin{bmatrix}\beta \\ \alpha\end{bmatrix} + (\alpha\hat{\boldsymbol{\tau}} + \epsilon)$. You can assume that $plim\left(\frac{1}{n}\begin{bmatrix}\mathbf{X}' \\ \hat{\mathbf{r}}'\end{bmatrix}\begin{bmatrix}\mathbf{X} & \hat{\mathbf{r}}\end{bmatrix}\right)^{-1}$ is a "well-behaved" finite-valued matrix.)

Part (c)
Now consider a "naive" first-stage regression that only uses the actual instruments on the right hand side. You can write it as

$$\mathbf{r} = \mathbf{Z}\delta + \psi \tag{3}$$

Assume that $plim\left(\frac{1}{n}\mathbf{Z}'\psi\right) = \mathbf{0}$. Express the fitted value of $\mathbf{r}$, call it $\hat{\mathbf{r}}$, in terms of $\mathbf{Z}$ and $\mathbf{r}$. Denote the residuals from this first-stage regression as $\hat{\hat{\psi}}$ and show that:

1. $\hat{\mathbf{r}}'\hat{\hat{\psi}} = 0$
2. $plim\left(\frac{1}{n}\hat{\mathbf{r}}'\epsilon\right) = 0$.

You can assume that $plim\left(\frac{1}{n}\mathbf{r}'\mathbf{Z}\right)$ and $plim\left(\left(\frac{1}{n}\mathbf{Z}'\mathbf{Z}\right)^{-1}\right)$ are "well-behaved" finite-valued matrices.

Part (d)
Now consider the second stage of this "naive 2SLS" estimator introduced in part (c) above, where $\mathbf{y}$ is regressed against $\mathbf{X}$ and the fitted value of $\mathbf{r}$.

As before, express $\mathbf{r}$ as the sum of population projection and residuals, i.e. $\mathbf{r} = \hat{\mathbf{r}} + \hat{\hat{\psi}}$, plug this expression into equation (1) of part (a), and write the model in partitioned form as shown in the hint to part (b). In this case, which additional *plim* conditions involving $\mathbf{X}$ must hold for the second stage to produce consistent estimates? Are these conditions likely to hold in practice?

In general, what does this tell you about the importance of including the exogenous variables from the original model in the first stage regression equation?

You can assume that $plim\left(\frac{1}{n}\begin{bmatrix}\mathbf{X}' \\ \hat{\hat{\mathbf{r}}}'\end{bmatrix}\begin{bmatrix}\mathbf{X} & \hat{\mathbf{r}}\end{bmatrix}\right)^{-1}$ is a "well-behaved" finite-valued matrix.

Question 4: A representative agent model with quadratic utility (45 minutes)

We consider a discrete-time representative agent model with a constant population. The lifetime expected utility of a household at time zero is written as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(c_t - bc_t^2\right)$$

We assume that b is small enough that marginal utility is always positive. Output y_t is produced by capital k_t subject to a productivity shock e_t:

$$y_t = Rk_t + e_t, \qquad R\beta = 1$$

The term R is the marginal product of capital. Capital accumulates through saving, and there is no depreciation:

$$k_{t+1} = y_t - c_t$$

In each period t, only current and past shocks $e_t, e_{t-1}, \ldots$ are known. The agent knows that the shock follows an autoregressive model:

$$e_t = \rho e_{t-1} + u_t, \qquad u_t \sim i.i.d.\, N(0,\sigma^2), \qquad -1 < \rho < 1$$

a) Write down the Euler equation for consumption. Does consumption follow a random walk?

b) You are told that the policy function for consumption has the form:

$$c_t = A + Bk_t + De_t$$

Solve out the constants A, B, D, using the "guess and verify" method.

c) Explain why σ^2, the measure of income uncertainty, does not affect consumption.

d) What is the unconditional variance of $c_{t+1} - c_t$, the change in consumption? How does an increase in ρ affect the unconditional variance? Is the variance the same as σ^2?

e) Suppose the productivity shock follows a second-order process:

$$e_t = 0.8e_{t-1} + 0.1e_{t-2} + u_t, \qquad u_t \sim i.i.d.\, N(0,\sigma^2)$$

Explain why the policy function will have the form:

$$c_t = A + Bk_t + D_1 e_t + D_2 e_{t-1}$$

You do not need to solve the model again, but you need to briefly explain your answer.

Problem 5: Macroeconomics (two parts, 1 and 2) 45 minutes

[1] Consider a two-period, two-generation economy. Each member of the time 1 generation has an endowment of e_1 and consumes and saves from this endowment. This generation dies at the end of time 1 and each member generates 1 offspring, a member of the time 2 generation. Each member of the time 1 generation cares about the consumption of its offspring. Each member of the time 2 generation has an endowment of e_2.

A time 1 parent can save by buying bonds on the international market. One unit of bonds will pay 1 unit of output at time 2, the real interest rate is fixed at r=0.

A time 1 parent wishes to maximize

$$ln(c_1)+ln(c_2).$$

in which c_1 is his own consumption and c_2 is the time 2 consumption of his offspring.

The budget constraints are

$$\text{Generation 1: } c_1+s_1+\tau_1=e_1,\ s_1 \geq 0.$$
$$\text{Generation 2: } c_2 = s_1 + e_2 - \tau_2$$

In these equations s_1 is the saving of the time 1 parent (which is constrained to be non-negative); and τ_i is time i lump-sum taxes.

The government has a fixed amount of spending at time 1. It can finance this spending by taxing the time 1 generation or by borrowing at time 1 (at r=0) and taxing the time 2 generation to pay off the principal and interest at time 2. Its budget constraint is

$$g = \tau_1 + \tau_2 .$$

Government spending yields benefits to neither generation.

The time 1 parent selects s_1 (and hence c_1) to solve:

$$\underset{s_1}{Max}\ ln(c_1)+ln(c_2), \text{ s.t. } s_1 \geq 0, \text{ and}$$

$$c_1=e_1 - s_1 - \tau_1;\ c_2 = s_1 + e_2 - \tau_2 .$$

(a) Solve the individual's optimization problem. Determine and explain when optimal saving is s_1=0 or s_1>0. Determine optimal consumption in each case and explain the economic intuition underpinning your answers.

(b) Suppose initially $\tau_1 = \tau_2$, and $e_1 > e_2$. What are s_1 and c_1 in this case? Suppose there is a marginal *decrease* in τ_1 what happens to c_1 and s_1? (Be sure to take account of how τ_1 and τ_2 are related via the government's budget constraint.) Does the Ricardian Equivalence Theorem hold in this case?

(c) Suppose initially $\tau_1 = \tau_2$, and $e_1 < e_2$. What are s_1 and c_1 in this case? Suppose there is a marginal *decrease* in τ_1 what happens to c_1 and s_1? What are the implications of your results, if any, for the Ricardian Equivalent Theorem?

[2] *A social security scheme.* In contrast to the setup in [1], assume the members of the time 1 generation can use government to pay themselves benefits at the expense of the time 2 generation. This is to say, the time 1 generation is free to choose g so to increase c_1.

The new budget constraints:

Generation 1: $c_1 + s_1 = e_1 + g,\ s_1 \geq 0,\ g \geq 0.$

Generation 2: $c_2 = s_1 + e_2 - \tau_2$

$g = \tau_2.$

The time 1 generation selects s_1 and g to

$$\underset{s_1, g}{Max}\ ln(c_1) + ln(c_2), \text{ s.t. } s_1 \geq 0,\ g \geq 0, \text{ and}$$

$$c_1 = e_1 + g - s_1;\ c_2 = s_1(1+r) + e_2 - \tau_2.$$

(a) Under what conditions will the time 1 generation select a positive g? What is the optimal value for g in this case? What is saving in this case?

(b) Suppose the optimal g is g>0. What are the effects on optimal g, c_1 and c_2 of a marginal change in e_2? Explain.