Departments of Economics and of Agricultural and Applied Economics

Ph.D. Qualifying Exam January 2014

PART I

January 13, 2014

Please answer all 3 questions.

Notice the time allotted to each question.

Problem 1 (60 minutes)

Consider the case of two commodities 1 and 2 and k consumers, each with utility function $U(x_1, x_2) = Ax_1 - \frac{1}{2}x_1^2 + x_2$ for $(x_1, x_2) \in \mathbb{R}^2_+$ and income $m > 0$, with $A > 0$. Commodity prices are $p_1 \geq 0$ and $p_2 = 1$.

(a) For a single consumer, determine the solution (x_1^*, x_2^*) of the problem

$$\max U(x_1, x_2) \text{ subject to } p_1 x_1 + p_2 x_2 = m.$$

Assume that m is sufficiently large so that $x_2^* > 0$ and x_1^* is of the form

$$x_1^* = x_1(p_1). \tag{1}$$

(b) Suppose there is a monopolist that produces commodity 1. The monopolist charges a price $p_1 \geq 0$. It has marginal cost c with $A > c \geq 0$. It faces the market demand function

$D(p_1) = k \cdot x_1(p_1)$ for $p_1 \geq 0$,

with the individual demand function $x_1(p_1)$ obtained in (a).

DETERMINE the monopolist's profit maximizing price p_1^* and its maximum profit π^* in terms of A, c and k.

(c) Suppose $c = 0$ and that A depends on the monopolist's advertising effort: $A = k + A_1$ if the monopolist incurs the advertising cost A_1^3, with $A_1 \geq 0$. Given π^* from (b), determine the monopolist's advertising level A_1^* that maximizes

$$\pi^* - A_1^3. \tag{2}$$

Give the maximum value of (2) in terms of k.

Problem 2 (60 minutes)

Consider an asymmetric Cournot oligopoly with n firms. The inverse demand function is

$$P(Q) = \begin{cases} a - Q = a - \sum_{i=1}^{n} q_i & \text{if } Q \leq a \\ 0 & \text{otherwise,} \end{cases}$$

where $a > 0$. In addition, assume that firm i's cost function is $C_i(q_i) = \frac{1}{2}c_i q_i^2$, with $0 < c_1 \leq ... \leq c_n$.

We are interested in determining the Cournot-Nash equilibrium of this model, i.e. the output profile $(q_1^*, ..., q_n^*) \in \mathbb{R}_+^n$ such that

$$\pi_i(q_i^*, q_{-i}^*) = \max_{q_i \geq 0} \overbrace{[P(q_i + Q_{-i}^*)q_i - \frac{1}{2}c_i q_i^2]}^{\pi_i(q_i, q_{-i}^*)}, \quad \forall\, i \in \{1, ..., n\},$$

where $q_{-i}^* = (q_1^*, ..., q_{i-1}^*, q_{i+1}^*, ..., q_n^*)$ and $Q_{-i}^* = \sum_{j \neq i} q_j^*$.

(i) Claim that, in the equilibrium, each of the n first-order conditions (FOC) holds with equality. (**HINT.** First, argue that we necessarily have $Q^* < a$; second, show that if $0 \leq Q_{-i}^* < a$, then $0 < q_i^* < a - Q_{-i}^*$.)

(ii) Write the FOC of firm i's problem and form the system (of n equations with n unknowns) that defines the equilibrium.

(iii) Rewrite firm i's FOC in the form $q_i^* + \alpha_i Q^* = \beta_i$, where α_i and β_i depend on the parameters of the model.

(iv) Determine Q^* and then derive the equilibrium outputs q_i^* $(i = 1, ..., n)$.

(v) Rank these equilibrium outputs by quantity. (Justify your answer.)

Problem 3 (60 minutes)

Consider a pure exchange economy $\mathcal{E} = \{(X^i, \succsim^i)_{i=1}^I, (\omega^i)_{i=1}^I\}$ with two consumers $i \in \{1, 2\}$ and two commodities $l \in \{1, 2\}$. Consumer i chooses among commodity bundles according to i's preferences $\succsim^i$ on $X^i = \mathbb{R}^2_+$ represented by the Leontief utility function:

$$u(x_1^i, x_2^i) = \min\{2x_1^i, x_2^i\}.$$

The initial endowments are $\omega^1 = (2, 10)$ and $\omega^2 = (10, 2)$. Let $p \in \mathbb{R}^2_+$, $p \neq 0$, be a price vector.

(a) Draw the Edgeworth-Box for the economy $\mathcal{E}$. Characterize the set of all Pareto-optimal allocations in the economy $\mathcal{E}$. Illustrate the set in the Edgeworth-Box.

(b) Determine consumer i's Walrasian demand correspondence $x^i(p, \omega^i)$. Determine the Walrasian equilibrium for the economy $\mathcal{E}$.

(c) Which of the Pareto-optimal allocations can be supported by a price equilibrium with transfers? Determine the price vector p, the distribution of wealths m^1 and m^2, and transfers T^1 and T^2.

Suppose that the consumers have an access to a firm. Consumer i's share of the firm is denoted by $\theta_1^i \in [0, 1]$ where $\sum_{i=1}^2 \theta_1^i = 1$. Suppose that the firm produces commodity 2 by using commodity 1. The technology of the firm is given by:

$$Y = \{(-y_1, y_2) \in \mathbb{R}^2 : y_2 \leq b \cdot y_1, 0 \leq y_1\}, \quad \text{where} \quad 10 \geq b > 0.$$

(d) Derive the firm's supply correspondence $y(p)$ as well as the profit function $\pi(p)$.

(e) Determine the Walrasian equilibrium for the private ownership economy.

Question 1 (30 minutes)

1. "In estimation, the method of Maximum Likelihood (ML) is based on the idea that the most representative estimate is the value of the parameter(s) that have the highest likelihood of giving rise to the observed data." Discuss.

2. Consider the specific case of the simple Normal model:

$X_t \backsim \mathsf{NIID}(\mu, \sigma^2), \;\; t = 1, 2, ..., n, ...$

where 'NIID' stands for 'Normal, Independent and Identically Distributed', σ^2 is assumed known and $f(x;\mu)=\frac{1}{\sigma\sqrt{2\pi}}\exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}, \;\; \mu\in\mathbb{R}, \;\; x\in\mathbb{R}$.

(a) Derive the distribution of the sample and the likelihood function.

(b) Derive the Maximum Likelihood Estimator $\widehat{\mu}_{MLE}$ of μ and show that it is indeed a maximum for the likelihood function.

(c) Derive the mean and variance of $\widehat{\mu}_{MLE}$.

(d) Derive the Fisher information and show how it relates to $Var(\widehat{\mu}_{MLE})$ and the Cramer-Rao lower bound.

(e) In light of your answers in (c)-(d), derive the sampling distribution of $\widehat{\mu}_{MLE}$ and state its finite sample properties.

(f) State the asymptotic properties of $\widehat{\mu}_{MLE}$.

Question 2 (20 minutes)

Consider a latent variable model where the observed outcome, y_i, depends on the latent variable c_i such that:

$$y_i = 1 \text{ if } c_i > 0, \quad \text{and} \quad y_i = 0 \text{ if } c_i \leq 0,$$
$$\text{where } c_i = X_i\beta + u_i, \text{ with } (u_i \mid X_i) \sim N(0, \kappa^2) \text{ and } \kappa^2 \neq 1.$$

The vector X_i contains variables that are observed by the econometrician. The scalar u_i represents the composite effect of variables that affect behavior but are not observed by the econometrician.

a) Assume that all *X*s are exogenous (i.e., there are no correlations between *X*s and *u*). Demonstrate that, in spite of this fact, standard probit estimation of this model will not produce a consistent estimator for β. [Note: You do not need to explicitly solve for the estimator to show this]

b) Now assume that $\kappa^2=1$, but there is one omitted variable *Z* that is independent of all *X*s in the model. Assume the omitted variable *Z* has a normal distribution with mean 0 and variance of τ^2, and that its marginal effect on c_i is γ. Demonstrate that under this neglected heterogeneity, a standard probit model still cannot produce a consistent estimator for β. [Note: You do not need to explicitly solve for the estimator to show this]

c) Do these identification problems make the standard probit model unfit for any meaningful inference? Why or why not? Explain.

Question III (40 minutes)

Consider the following system of equations:

$$\mathbf{y}_1 = \mathbf{X}_1\boldsymbol{\beta}_1 + \beta_2\mathbf{y}_2 + \boldsymbol{\epsilon}_1 \tag{1}$$

$$\mathbf{y}_2 = \mathbf{X}_2\pi + \boldsymbol{\epsilon}_2 \tag{2}$$

where all matrices and vectors are of length n, and the column dimensions of $\mathbf{X}_1$ and $\mathbf{X}_2$ are k_1 and k_2, respectively, with $k_2 > k_1$. Your main goal is to consistently estimate $\boldsymbol{\beta} = \begin{bmatrix}\boldsymbol{\beta}_1' & \beta_2\end{bmatrix}'$, but you are concerned that the two error terms are correlated, making $\mathbf{y}_2$ an endogenous regressor in the first equation.

Your strategy is to use the OLS fitted values from the second equation (call them $\hat{\mathbf{y}}_2$) as instrument for $\mathbf{y}_2$ in a Two-Stage least Squares (TSLS) estimation of the first equation.

Throughout you can assume that $\mathbf{X}_1$ is orthogonal to $\mathbf{X}_2$ and $\mathbf{y}_2$, and that neither $\mathbf{X}$ matrix is correlated with any of the error terms.

Part (a)
Denote the residuals from the OLS estimation of the second equation as $\mathbf{e}_2$. Show that they are orthogonal to $\mathbf{X}_1$. Show that the fitted values $\hat{\mathbf{y}}_2$ are also orthogonal to $\mathbf{X}_1$

Part (b)
Show that $\hat{\mathbf{y}}_2'\hat{\mathbf{y}}_2 = \mathbf{y}_2'\hat{\mathbf{y}}_2 = \hat{\mathbf{y}}_2'\mathbf{y}_2$. (This will be needed later).

Part (c)
Let $\mathbf{X} = \begin{bmatrix}\mathbf{X}_1 & \mathbf{y}_2\end{bmatrix}$ and $\mathbf{Z} = \begin{bmatrix}\mathbf{X}_1 & \hat{\mathbf{y}}_2\end{bmatrix}$.
Describe (in words or math) the regression model for the *first stage* of the TSLS procedure. Let $\hat{\mathbf{X}}$ be the fitted matrix from this model, and show that $\hat{\mathbf{X}} = \begin{bmatrix}\mathbf{X}_1 & \hat{\mathbf{y}}_2\end{bmatrix} = \mathbf{Z}$

Part (d)
Describe, in words, the second stage of the TSLS procedure and solve for $\hat{\boldsymbol{\beta}}_{TSLS}$ in terms of $\mathbf{X}_1$,$\hat{\mathbf{y}}_2$, and $\mathbf{y}_1$.

Part (e)
Show that this TSLS approach for the estimation of $\boldsymbol{\beta}$ is equivalent to adding the second-equation residuals $\mathbf{e}_2$ to equation (1) as an additional regressor and using OLS, i.e. by estimating

$$\mathbf{y}_1 = \mathbf{X}_1\boldsymbol{\beta}_1 + \beta_2\mathbf{y}_2 + \gamma\mathbf{e}_2 + \boldsymbol{\nu}_1 = \mathbf{X}\boldsymbol{\beta} + \gamma\mathbf{e}_2 + \boldsymbol{\nu}_1 \tag{3}$$

where $\mathbf{X}$ and $\boldsymbol{\beta}$ are defined as above.

Call this estimator $\tilde{\mathbf{b}}$.
(Hint: Use partitioned regression results. Start directly with the partitioned regression solution for $\tilde{\mathbf{b}}$, which will include a residual-maker matrix. Call that matrix $\mathbf{M}_e$. Note that $\mathbf{M}_e\mathbf{y}_2 = \hat{\mathbf{y}}_2$.)

Part 2: Macroeconomics, Two Questions
Macro Question 1 45minutes

Consider a modified Malthusian model of population growth. There is a fixed resource, call it land, L. Assume, however, there is technological progress as illustrated below.

The time t population is N_t. Population growth is described by $N_{t+1} = N_t n_t$, in which n_t is the number of offspring each member of generation t chooses to rear. Total output at time t is

$$Y_t = (A_t L)^{\alpha} N_t^{1-\alpha}, \quad A_{t+1} = A_t(1+g).$$

Time t output is spread evenly among members of generation t. Let $\hat{x}_t \equiv (A_t L / N_t)$, then time t per capita output can be written as

$$y_t \equiv Y_t / N_t = (A_t L / N_t)^{\alpha} \equiv \hat{x}_t^{\alpha}.$$

Each member of the time t generation solves the utility maximization problem:

$$\max_{c,n} \ \ln(c_t) + \ln(n_t), \ s.t. \ c_t + kn_t = y_t \equiv \hat{x}_t^{\alpha}.$$

(a) Solve an individual's optimization problem, determining c_t and n_t as functions of $y_t = \hat{x}_t^{\alpha}$.

(b) What is the steady state value of $\hat{x}$?

(c) Given your answer in (b), determine the steady state values for c_t and y_t. What is the (gross) steady state growth rate of the population, n_t? Despite the presence of technological progress, in what sense is this still a Malthusian model?

(d) Why does technological progress not yield a demographic transition in this model?

At time 0 the economy is its original steady state; however, new farming techniques open new arable land, that is, at time 0 there is an increase in L. Answer (e) and (f).

(e) What happens to y_0? What happens to steady state y? Show the time path of y_t from time 0 forward.

(f) What happens to the time 0 population growth rate? What happens to its steady state value. Show the time path on n_t from time 0 forward.

Macro Question 2 **45 minutes**

Money Neutrality, or Not

The following equations describe the macro-economy. All variables are in log.

Aggregate demand: $m_t + v_t = p_t + y_t$

Price setting: $p_t = w_t + y_t$

Wage setting: $w_t = E_{t-1}(p_t)$

Demand shock: $v_t = v_{t-1} + e_t$, where e_t has mean zero and is uncorrelated across time.

The money supply m_t is exogenously determined, and m_t is set based on information up to time t-1, that is,

$$m_t = \sum_{i=1}^{\infty} b_i e_{t-i} .$$

(a) Does the model imply that money affects output? Explain. (35 points)

(b) Briefly mention one major difference between this model and the standard New Keynesian model. (10 points)

(c) Suggest a way of changing the <u>wage setting equation</u> so that money affects output. Explain the intuition behind the change. Show how monetary policy should be set to minimize the variance of output. (35 points)

(d) Suppose v_{t-1}=0 and there is now one-time shock e_t=1 (and e_i=0, for i>t). Under the setup in (a), for $i \geq t$, describe how y_i and p_i are affected. (20 points)

Note: the listed points indicate the relative weights given each part of question 2.