Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, August 2019

Part 1: Microeconomics

4 Questions, 4 pages

Note: The minutes assigned to each question indicate the weight given to the question. For example, question 1 is 30 minutes out of a total of 180 minutes and thus counts for 3/18 of the grade for this exam.

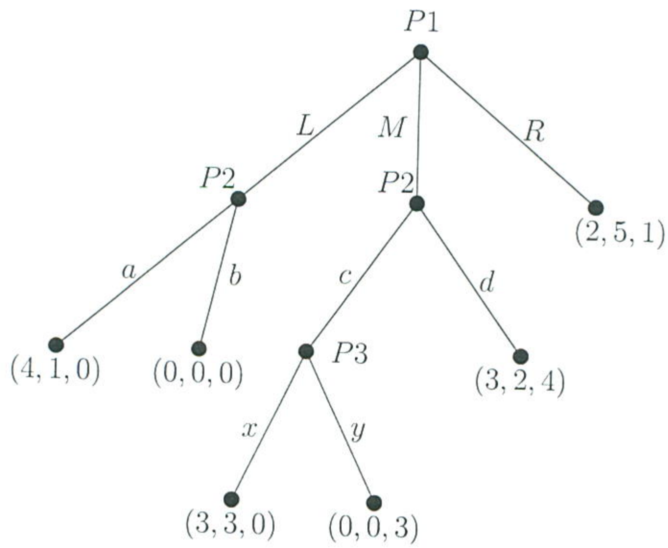
Problem 1 (30 min). Suppose $X = \mathbb{R}^2_+$ and $\mathscr{A} = \{\mathbb{B}(\mathbf{p}, m) | \mathbf{p} = (p_1, p_2) \gg 0 \text{ and } m > 0\}$. Suppose a choice function $C : \mathscr{A} \to X$ satisfies WARP. Suppose that C is strictly monotonic; that is, $C(\mathbb{B}(\mathbf{p}, m)) \neq (x_1, x_2)$ if there is $(x_1', x_2') \in \mathbb{B}(\mathbf{p}, m)$ such that $(x_1', x_2') > (x_1, x_2)$.

a) Write down the definition of WARP for C on $\mathscr{A}$.

b) Suppose $C(\mathbb{B}((5,5), 100)) = (10, 10)$. Then is it possible to have $C(\mathbb{B}((6,6), 180)) = (0, 30)$?

c) Suppose $C(\mathbb{B}((1,3), 60)) = (18, 14)$ and $C(\mathbb{B}((3,1), 60)) = (12, 24)$. Determine all possible $C(\mathbb{B}((2,2), 80))$.

d) Suppose $C(\mathbb{B}((p_1, p_2), m)) = (x_1, x_2)$ and $C(\mathbb{B}((p_1, p_2 - \epsilon), m - x_2\,\epsilon)) = (x_1', x_2')$ for some $\epsilon \in (0, p_2)$. Show that $x_2 \leq x_2'$.



Problem 2 (40 min). Consider the above dynamic game with three players, P1, P2, and P3.

a) For each player $i \in \{1, 2, 3\}$, write down the set of strategies of player i.

b) Find all (pure strategy) SPNE of this game.

c) Find all (pure strategy) NE of that game.

Problem 3 (50 min). Two players are bargaining over a pie of initial size $s = 1$. Player 1 moves first by offering a portion $x \in [0, 1]$ to Player 2. If this first offer is accepted, the game ends and the payoffs are $1 - x$ for Player 1 and x for Player 2. To capture the effects of discounting, assume that the pie is subject to decay and, in case Player 2 rejects the first offer, the residual value of the pie is $\delta \in (0, 1)$. Following her refusal, Player 2 has to make a counteroffer by proposing an amount $y \in [0, \delta]$ to player 1. If player 1 accepts the counteroffer, she gets y and Player 2 gets $\delta - y$. Otherwise, the players both get 0. Each person cares only about the amount she receives, and would like to receive as much as possible.

a) Draw a (schematic) game-tree representation of this bargaining process.

b) Give a Nash equilibrium that is not subgame perfect (justify your answer).

c) Use backward induction to derive a **subgame perfect Nash equilibrium** (clearly specify the strategies and the outcome).

d) Describe the **subgame perfect outcome** for the variant where the two players are allowed to repeat the above bargaining process up to $T \geq 2$ times (as long as an offer has not been accepted), with the pie losing a fraction $1 - \delta$ of its value following every refusal (by either player). If all $2T$ offers have been rejected then the game ends and the players each get 0. *What is the limit of this equilibrium outcome as T goes to infinity?*

Problem 4 (60 min). Let $\mathcal{E} = \{(X^i, \succsim^i)_{i=1}^I, \omega\}$ be a pure exchange economy with two consumers $i \in \{1,2\}$ and two commodities $l \in \{1,2\}$. Consumer 1's initial endowment is $\omega^1 = (0,0)$. Consumer 2's initial endowment is $\omega^2 = (10,10)$. Each consumer has preferences $\succsim^i$ defined on the set of commodity bundles $X^i = \mathbb{R}^2_+$. Consumer 1's preferences have the lexicographic form, i.e., for any $x^1, y^1 \in \mathbb{R}^2_+$, $(x^1_1, x^1_2) \succsim^1_{lex} (y^1_1, y^1_2)$ if either $x^1_1 > y^1_1$ or $x^1_1 = y^1_1$ and $x^1_2 \geq y^1_2$. Consumer 2's has Cobb-Douglas preferences represented by the utility function: $u^2(x^2_1, x^2_2) = \sqrt{(x^2_1)}\sqrt{(x^2_2)}$.
Let $p \in \mathbb{R}^2_+$, $p \neq 0$, be a price vector.

(a) Are preferences of Consumer 1 continuous? (Prove or disprove it).

(b) Compute i's Walrasian demand correspondence $x^i(p, \omega^i)$ for each $p \in \mathbb{R}^2_+$.

(c) Draw the Edgworth box for the economy $\mathcal{E}$. Determine (graphically and formally) the set of all Pareto optimal allocations. What is the set of core allocations?

(d) Determine the Walrasian equilibrium for $\mathcal{E}$.

(e) Which of the Pareto optimal allocations (determined in Part (b)) can be generated by a price equilibrium with transfers (PET)? Determine the price vector p, the distribution of wealth levels m^1 and m^2, as well as the transfers T^1 and T^2. Note, $T^1 + T^2 = 0$.

Consider now endowments $\omega^1 = (\omega^1_1, \omega^1_2) \in \mathbb{R}^2_+$ and $\omega^2 = (\omega^2_1, \omega^2_2) \in \mathbb{R}^2_+$ such that $\omega^1_1 + \omega^2_1 = 10$ and $\omega^1_2 + \omega^2_2 = 10$.

(f) Determine all Walrasian equilibria for $\mathcal{E}$ as a function of the initial endowments.

(g) Determine all core allocations for $\mathcal{E}$ as a function of the initial endowments.

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Part 2: Econometrics and Macroeconomics

Part 2A: Econometrics (3 Questions on 4 pages, 20 minutes each)
Part 2B: Macroeconomics (1 hour, 1 Question on 2 pages)

Question 1 (20 minutes)

(a) Consider the simple Normal model:

$$X_t \backsim \text{NIID}(\mu, \sigma^2),\ t{=}1, 2, ..., n, ...,$$

with σ^2 is known; 'NIID' stands for 'Normal, Independent and Identically Distributed'.

(i) State the sampling distributions of the test statistic:

$$d(\mathbf{X}){=}\frac{\sqrt{n}(\overline{X}_n-\mu_0)}{\sigma},\quad \overline{X}_n{=}\frac{1}{n}\sum_{t=1}^{n} X_t,$$

under the null for the hypotheses:

$$H_0\text{: } \mu{=}\mu_0, \text{ vs. } \quad H_1\text{: } \mu > \mu_0. \tag{1}$$

(ii) Explain what is needed for one to define an optimal test for (1).

(b) (i) Compare and contrast the sampling distributions of $d(\mathbf{X})$ under H_0 and H_1 to that of the pivotal function $d(\mathbf{X};\mu)$:

$$d(\mathbf{X};\mu){=}\frac{\sqrt{n}(\overline{X}_n-\mu)}{\sigma} \overset{\mu=\mu^*}{\backsim} \mathsf{N}(0,1), \tag{2}$$

where μ^* denotes the 'true' value of μ, whatever that happens to be.

(ii) Explain how (2) can be used to construct an $(1{-}\alpha)$ Confidence Interval for μ.

(c) (i) Explain why the sampling distribution of $d(\mathbf{X})$ under $\mu_1 > \mu_0$ takes the form:

$$d(\mathbf{X}){=}\frac{\sqrt{n}(\overline{X}_n-\mu_0)}{\sigma} \overset{\mu=\mu_1}{\backsim} \mathsf{N}(\delta_1,1),\ \delta_1{=}\frac{\sqrt{n}(\mu_1-\mu_0)}{\sigma},\ \text{for all } \mu_1{>}\mu_0. \tag{3}$$

(ii) Using your answer in (a)(i) and the distribution in (3) to define the notions of (I) type I error probability, (II) type II error probability, (III) the p-value, and (IV) compare and contrast (I) and (III).

(d) State the fallacies of acceptance and rejection and explain why the accept/reject rules and the p-value are vulnerable to these fallacies when they are interpreted as providing evidence for or against the null or the alternative.

Question 2 (20 minutes)

Assume you collect samples of equal size n for each of two groups, female (F) and male (M). For each group you observe an n by 1 outcome variable $\mathbf{y}_j$, $j = F, M$, and k explanatory variables collected in n by k matrix $\mathbf{X}_j$, $j = F, M$.

Assume that in truth, outcomes for each group follow a *separate CLRM* with the typical error properties and *common variance*, that is:

$$\begin{aligned} \mathbf{y}_F &= \mathbf{X}_F \boldsymbol{\beta}_F + \boldsymbol{\epsilon}_F \\ \mathbf{y}_M &= \mathbf{X}_M \boldsymbol{\beta}_M + \boldsymbol{\epsilon}_M \\ \boldsymbol{\epsilon}_F &\sim n\left(\mathbf{0}, \sigma^2 \mathbf{I}_n\right), \quad \boldsymbol{\epsilon}_M \sim n\left(\mathbf{0}, \sigma^2 \mathbf{I}_n\right) \\ \boldsymbol{\beta}_F &\neq \boldsymbol{\beta}_M \end{aligned} \tag{1}$$

Part (a)

a Write down the OLS estimator for each model - call them $\mathbf{b}_F$ and $\mathbf{b}_M$, respectively.

b Show that they are unbiased for their corresponding true parameter vectors.

Part (b)

a Consider an *equivalent* single regression model that stacks the two vectors of dependent variables into $\tilde{\mathbf{y}} = \left[\mathbf{y}_F' \quad \mathbf{y}_M'\right]'$:

$$\tilde{\mathbf{y}} = \tilde{\mathbf{X}} \tilde{\boldsymbol{\beta}} + \tilde{\boldsymbol{\epsilon}}. \tag{2}$$

Show the explicit form of $\tilde{\mathbf{X}}$, $\tilde{\boldsymbol{\beta}}$, and $\tilde{\boldsymbol{\epsilon}}$ in terms of the original model components in equ. (1). Recall that the model has to allow for separate coefficient vectors for the two groups.

b Show that the OLS solution for this model, call it $\tilde{\mathbf{b}}$, is equivalent to that obtained from estimating the two group-specific models separately.

Part (c)

Now consider a (wrong) model that assumes equal coefficient effects on both groups. Let the model be written as:

$$\tilde{\mathbf{y}} = \mathbf{X} \boldsymbol{\gamma} + \boldsymbol{\nu}, \tag{3}$$

where, as before, $\tilde{\mathbf{y}} = \left[\mathbf{y}_F' \quad \mathbf{y}_M'\right]'$, and $\boldsymbol{\gamma}$ is k by 1.

a Show the explicit form for $\mathbf{X}$.

b Given the form of the *correct* single model derived in the previous part, show the explicit contents of the error vector $\boldsymbol{\nu}$.

c Show that this error vector is correlated with $\mathbf{X}$, i.e. that $E\left(\mathbf{X}'\boldsymbol{\nu}\right) \neq \mathbf{0}$.

d What does this imply for the OLS estimate of $\boldsymbol{\gamma}$ (call it $\hat{\boldsymbol{\gamma}}$)?

e Under what condition would the correlation between $\mathbf{X}$ and $\boldsymbol{\nu}$ go to zero?

Question 3 (20 minutes)

You are involved in a research project on beach visitation in Florida (FL). Beach visitors can be divided into three groups: (1) Locals (FL residents who live within 60 miles of a given beach), (2) FL tourists (FL residents who live further away than 60 miles from a given beach), and (3) out-of-state tourists. You are interested in the true proportions of these groups for all visitors to *Siesta Key*, a large, popular beach, on a specific day. Let these true proportions be labeled as π_1, π_2, and π_3, for locals, FL tourists, and out-of-state tourists respectively. Naturally, $\sum_{j=1}^{3} \pi_j = 1, \quad j = 1 \ldots 3$. Also, let $\boldsymbol{\pi} = \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix}'$.

On your day of interest, you randomly sample n visitors to Siesta Key. For each, you write down an indicator vector $\mathbf{z}_i$ that shows to which group the person belongs. For example, if the person is a local, then $\mathbf{z}_i = \begin{bmatrix} z_{1i} & z_{2i} & z_{3i} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$. Similarly, if the person is a FL tourist, the second element in $\mathbf{z}_i$ will be "1" and the other two will be "0," and if the person is an out-of-state tourist, $z_{3i} = 1$, and $z_{1i} = z_{2i} = 0$. Let the total number of individuals sampled for each group be n_1, n_2, and n_3, respectively.

You stipulate that each individual indicator vector $\mathbf{z}_i$ follows a multinomial likelihood, given as:

$$p(\mathbf{z}_i|\boldsymbol{\pi}) = \left(\frac{1}{\prod_{j=1}^{3} z_{ji}!}\right) \prod_{j=1}^{3} \pi_j^{z_{ji}} = \prod_{j=1}^{3} \pi_j^{z_{ji}} \tag{1}$$

Part (a)
Write down the likelihood for the entire sample of n observations. You can label the entire set of n indicator vectors as $\mathbf{z}$. Simplify as much as possible.

Part (b)
As a prior for $\boldsymbol{\pi}$ you choose a Dirichlet distribution. The density and moments for the Dirichlet are given as:

$$\begin{aligned} p(\boldsymbol{\pi}) &= \left(\frac{\Gamma(\tilde{\alpha})}{\prod_{j=1}^{3} \Gamma(\alpha_j)}\right) \prod_{j=1}^{3} \pi_j^{\alpha_j - 1}, \quad \alpha_j > 0, \forall j, \\ E(\pi_j) &= \frac{\alpha_j}{\tilde{\alpha}}, \quad V(\pi_j) = \frac{\alpha_j(\tilde{\alpha} - \alpha_j)}{\tilde{\alpha}^2(\tilde{\alpha} + 1)}, \quad \text{where} \\ \tilde{\alpha} &= \sum_{j=1}^{3} \alpha_j \end{aligned} \tag{2}$$

a Derive the *kernel* of the posterior distribution $p(\boldsymbol{\pi}|\mathbf{y})$, and determine the statistical distribution for the full posterior.

b Show the posterior parameters for this distribution (label them α_j^*, $j = 1 \ldots 3$).

Part (c)
Assume you have visitor information from *other nearby beaches*, with average proportions for the three visitor groups of 0.5, 0.1, and 0.4. Interpreting these averages as prior expectations, and

letting $\alpha_1 = 10$, derive the prior parameters α_2 and α_3, as well as the prior variances for the three shares. Round the variances to four decimals.

Part (d)
Assume your Siesta Key sample of 200 visitors produces $n_1 = 80$, $n_2 = 10$, and $n_3 = 110$.

a Using all the information from above, compute the posterior expectations and variances for the population shares. Round all expectations to three decimals, and all variances to four decimals.

b How can you tell that the collected data has brought information to the prior?

c The town of Siesta Key is willing to sponsor an advertising campaign targeted to *in-state tourists* (group 2), if the posterior share of this group falls below 10%. What will be the town's decision?

Macroeconomics 1 hour

Consider an overlapping generations model in which individuals live for three periods. There is no population growth, so at any time t there N young agents, N middle-aged agents and N retirees. The time t generation is young at time t, middle-aged at time t+1 and is retired at time t+2. Each generation produces one unit of output when young and one unit in middle-age. Output is perishable and cannot be stored for future use. In retirement an individual produces nothing. Assume there is a loans market with a constant gross interest rate of R (R=1+r, r being the real interest rate).

A time t young person has the following budget constraint:

$$(c_{1,t})+(c_{2,t})/R+(c_{3,t})/R^2 = 1+1/R. \quad (1)$$

In this equation $(c_{1,t})$ is generation t's consumption when young, $(c_{2,t})$ is consumption in middle age, and $(c_{3,t})$ is consumption in retirement.

Given the budget constrain in (1), a young, time t individual chooses $(c_{1,t})$, $(c_{2,t})$, and $(c_{3,t})$ to solve the following utility maximization problem:

$$\max \ln(c_{1t})+\ln(c_{2t})+\ln(c_{3t}),$$

(a) Determine an individual's optimal consumption triplet, $[(c_{1,t}), (c_{2,t}), (c_{3,t})]$.

(b) Determine an individual's net saving at each time of life, $(s_{1,t})$, $(s_{2,t})$, and $(s_{3,t})$.

(c) Show that R=1 is an equilibrium in this model. (Hint: Individuals are identical and at each time t the total saving must equal zero, $(s_{1,t})+(s_{2,t-1})+ (s_{3,t-2})=0$ (or equivalently, $(c_{1,t})+(c_{2,t-1})+(c_{3,t-2})=2$).

Population Growth

Modify the model to include a constant growth rate in the population. So the t+1 generation is $N_{t+1} = GN_t$, in which G is a constant greater than 1 (G=1+g, g>0).

(d) At any time t, what is total savings in this economy?

(e) At any time t, what is total consumption in this economy?

(f) Find an equilibrium interest rate for this economy.

The Model with Money

Suppose there the loans market does not exist, but each individual can save in the form of money. The nominal money supply is fixed a M. There is a monetary equilibrium that replicates the economy in (a)-(c). Let P_t be the time t price of output. The real return to money is P_t/P_{t+1}. To replicate the economy in (a)-(c), assume the price level is stable so that $P_t/P_{t+1}=1$, and $P_t=P$,a constant.

In the form of money (real money), a member of generation t's real saving is ϕ_{1t} when young, ϕ_{2t} in middle age, and ϕ_{3t} in retirement.

(g) At any time t the real demand for money must equal M/P, the real supply of money. Determine an equilibrium value for P. (Should I give a hint?)