Qualifying Exam Retake
Microeconomics Section
Fall 2019

Problem 1 (60 minutes)
A crime is observed by a group of $n \geq 2$ people. Each person would like the police to be informed but prefers that someone else make the phone call. Precisely, suppose that each person attaches the value v to the police being informed and bears the psychological cost c if she makes the phone call, where $v > c > 0$. In case nobody calls the police, each of the n people has a payoff of 0. The situation can be modeled as a strategic form game where each of the n players chooses between the two actions $\{Call, Don't\ call\}$.

(a) We are interested in finding the symmetric mixed-strategy Nash equilibrium. Let p be the probability with which each person calls in this symmetric profile.

(a.1) Argue that $p \in (0, 1)$ and then determine its value.

(a.2) Consider the two events: "person 1 does not call the police" and "nobody calls the police". Given the symmetric Nash equilibrium found above, determine the respective probabilities of these events. How do these probabilities change as n increases?

Examine now the variant where there are two groups of witnesses (e.g., students and teachers in a schoolyard). Every person still attaches the value v to the police being informed of the crime; and each witness of group $i = 1, 2$ bears the cost c_i if she makes the phone call (with $v > c_1 > c_2 > 0$). Assume that the number of witnesses in group i is $n_i \geq 1$ (with $n = n_1 + n_2$).

(b) Does a fully symmetric equilibrium (in mixed strategies) exist in this case? Justify your answer.

(c) Assuming its existence, compute a Nash equilibrium where every witness assigns a positive probability to each one of the two actions.

(d) How does the equilibrium individual probability (of calling the police) vary from group 1 to group 2? Interpret your result.

Problem 2 (60 minutes)

Suppose a preference relation $\succsim$ on $X = \mathbb{R}^2_+$ is rational, convex, and continuous.

(a) Show that the demand correspondence $\mathbf{x}(\mathbf{p}, m)$ is convex valued.

(b) Take any $x, y \in X$ with $x \succ y$. Show that the following set is closed.

$$W(x, y) = \{z \in X \,|\, \text{either } z \sim x \text{ or } y \succsim z\}.$$

(c) Sketch the set $W(x, y)$ in a diagram when $\succsim$ is in addition strictly monotone.

(d) Depict the set $W(x, y)$ in a diagram when $\succsim$ is represented by the utility function $U(z_1, z_2) = (z_1 - 4)^2 + (z_2 - 4)^2$, $x = (2, 2), y = (3, 3)$.

Problem 3 (60 minutes)

A finite pure exchange economy is specified by a tuple $\mathcal{E} = (X_i, \succsim_i, \omega_i)_{i \in I}$. The economy consists of a finite set I of consumers. There exist a finite number of commodities $l = 1, \dots, \ell$. Each consumer has consumption set $X_i = \mathbb{R}^\ell_+$. Superscripts denote commodities. Hence x^l_i stands for the quantity of commodity l consumed by $i \in I$. Consumer i's consumption bundles assume the form $x_i = (x^1_i, \dots, x^\ell_i)$. However, we write $p = (p_1, \dots, p_\ell)$ for price systems. In general, ℓ can be any finite number. In the following, we proceed with $\ell = 2$. Consumer i has complete and transitive preferences on X_i, represented by the binary relation $\succsim_i$. Finally, each consumer is endowed with a commodity bundle $\omega_i \in X_i$.

There are two countries, country A and country B, initially with separate economies.

Part A. Country A consists of two consumers 1 and 2, each with endowment bundle $\omega_i = (1, 1)$ and utility function $U_i(x^1_i, x^2_i) = (x^1_i)^2 + (x^2_i)^2$. Determine the competitive (Walrasian) equilibria of A's economy.

Part B. Country B consists of consumer 3 with endowment bundle $\omega_3 = (1, 1)$ and utility function $U_3(x^1_3, x^2_3) = x^1_3 + 2x^2_3$. Determine the competitive (Walrasian) equilibria of B's economy.

Part C. Now suppose the economies of countries A and B are integrated so that there is a common market with $I = \{1, 2, 3\}$. Your task is two-fold:

- First decide whether the integrated economy has a competitive equilibrium.
- Determine the competitive equilibria if the economy has one. Otherwise, prove that the economy does not have a competitive equilibrium.

Qualifying Re-take Examination

November 2019

Econometrics Portion

Question 1 (suggested time: 20 minutes)

(a) Consider the simple Normal model:

$$X_t \backsim \mathsf{NIID}(\mu, \sigma^2), \ \ t{=}1, 2, ..., n, ...,$$

with σ^2 is <u>known</u>; 'NIID' stands for 'Normal, Independent and Identically Distributed'.

(i) State the sampling distributions of the test statistic:

$$d(\mathbf{X}){=}\tfrac{\sqrt{n}(\overline{X}_n-\mu_0)}{\sigma}, \ \overline{X}_n{=}\tfrac{1}{n}\sum_{t=1}^{n} X_t,$$

under the null and under the alternative hypotheses:

$$H_0\!: \ \mu \leq \mu_0, \text{ vs. } \ \ H_1\!: \ \mu > \mu_0. \tag{1}$$

(ii) Define the optimal Neyman-Pearson (N-P) test in terms of $d(\mathbf{X})$ and explain what 'optimal' means in this case.

(b) (i) Compare and contrast the sampling distributions of $d(\mathbf{X})$ under H_0 and H_1 to:

$$d(\mathbf{X};\mu){=}\tfrac{\sqrt{n}(\overline{X}_n-\mu^*)}{\sigma} \overset{\mu=\mu^*}{\backsim} \mathsf{N}(0, 1), \tag{2}$$

where μ^* denotes the 'true' value of μ, whatever that happens to be.

(ii) Explain why (2) provides the relevant sampling distribution for constructing an $(1{-}\alpha)$ Confidence Interval (CI) for μ.

(c) Define and explain briefly the notions of (i) type I error probability, (ii) power of the test, (iii) the p-value, and (iv) compare and contrast (i) with (iii).

(d) State the fallacies of acceptance and rejection and explain why the accept/reject rules and the p-value are vulnerable to these fallacies when they are interpreted as providing evidence for or against the null or the alternative.

Question 2 (suggested time: 20 minutes)

Consider the following true population model for a given individual i:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 T_i + \beta_2 x_i + \epsilon_i \quad \text{with} \\ \epsilon_i &\sim n\left(0, \sigma^2\right), \end{aligned} \tag{1}$$

where y_i is some continuous outcome of interest, T_i is a binary (0/1) treatment indicator, x_i is a continuous explanatory variable, and ϵ_i is a standard error term with the usual CLRM properties.

Part (a), 4 points

a What is the true treatment effect?

b Show that it can be expressed as a difference between two expectations (conditional on x_i).

Part (b), 6 points

Assume you collect a random sample of individuals from this population. In your sample, you have n_1 treated and n_0 un-treated ("control") observations. For ease of notation, let outcome and explanatory variable for a treated observation be denoted as y_{Ti} and x_{Ti}, respectively. Analogously, let y_{Ci} and x_{Ci} be outcome and explanatory variable for a given control observation.

Assume you use some matching procedure to pair each treated observation with a single control observation. You then consider the following estimator for the population treatment effect (="average treatment effect for the treated"):

$$ATT_G|\mathbf{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} \left(y_{Ti} - y_{Ci}\right),$$

where subscript "G" stands for "generic," $\mathbf{x}$ collects all relevant x_i's, and the summation is over all treated observations.

a Assume that the average difference between x_{Ti} and x_{Ci} across all matched pairs equals $\delta \neq 0$. Show that, under this assumption, this generic ATT (given $\mathbf{x}$) is biased.

b Under what conditions would this bias go to zero? Provide some verbal intuition.

Part (c), 10 points

Now consider applying the linear regression model given in (1) to the *matched control observations*, that is:

$$\begin{aligned} y_{Ci} &= \beta_0 + \beta_2 x_{Ci} + \epsilon_i \quad \text{with} \\ \epsilon_i &\sim n\left(0, \sigma^2\right), \end{aligned} \tag{2}$$

a Assume this model produces unbiased estimates for β_0 and β_2 (after all, you used the correct functional specification, and the correct error assumptions...). Call the coefficient estimates $\hat{\beta}_0$ and $\hat{\beta}_2$, respectively. Consider the linear predictions flowing from this model plugging in either some x_{Ci} or some x_{Ti}. Call these predictions $\hat{y}_{Ci}$ and $\hat{y}_{Ti}$, respectively. Show that they are also unbiased for the corresponding $E\left(y_i|x_{Ti}\right)$ and $E\left(y_i|x_{Ci}\right)$, respectively.

b Now consider the *regression-adjusted* treatment effect estimator ATT_R, given as:

$$ATT_R|\mathbf{x} = \frac{1}{n_1}\sum_{i=1}^{n_1}\left((y_{Ti} - y_{Ci}) - (\hat{y}_{Ti} - \hat{y}_{Ci})\right),$$

Show that this estimator is unbiased, regardless of the (average) difference between x_{Ti} and x_{Ci}.

c In terms of unbiasedness how does this regression-adjusted matching estimator for the true treatment effect compare to directly estimating β_1 using the regression model in (1) and the entire sample of treated and controls? *(A verbal response is sufficient).*

Question 3 (suggested time: 20 minutes)

Part (a), 8 points

Consider the Bayesian estimation of the Classical Linear Regression Model with independent priors. When characterizing the conditional posterior kernel of the coefficient vector $\boldsymbol{\beta}$, conditional on error variance σ^2, we arrived at the following expression:

$$\begin{aligned} &p\left(\boldsymbol{\beta}|\sigma^2, \mathbf{y}, \mathbf{X}\right) \propto \\ &exp\left(-\tfrac{1}{2}\left(\boldsymbol{\beta}'\mathbf{V}_0^{-1}\boldsymbol{\beta} - 2\boldsymbol{\beta}'\mathbf{V}_0^{-1}\boldsymbol{\mu}_0\right)\right) * exp\left(\tfrac{1}{2}\boldsymbol{\mu}_0'\mathbf{V}_0^{-1}\boldsymbol{\mu}_0\right) * \\ &exp\left(-\frac{1}{2\sigma^2}\left(-2\boldsymbol{\beta}'\mathbf{X}'\mathbf{y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta}\right)\right), \end{aligned} \tag{1}$$

where $\boldsymbol{\mu}_0$ and $\mathbf{V}_0$ are the prior mean and variance of $\boldsymbol{\beta}$, respectively, $\mathbf{y}$ is the data vector of outcomes and $\mathbf{X}$ is the explanatory data matrix.

a Manipulate this expression to obtain the form

$$p\left(\boldsymbol{\beta}|\sigma^2, \mathbf{y}, \mathbf{X}\right) \propto exp\left(\boldsymbol{\beta}'\mathbf{V}_1^{-1}\boldsymbol{\mu}_1 - \tfrac{1}{2}\boldsymbol{\beta}'\mathbf{V}_1^{-1}\boldsymbol{\beta}\right), \tag{2}$$

where $\mathbf{V}_1$ and $\boldsymbol{\mu}_1$ are placeholders for more complex expressions. Show the explicit form of $\boldsymbol{\mu}_1$ and $\mathbf{V}_1$.

b Recognizing the expression in (2) as the kernel of a multivariate normal density, draw conclusions as to the exact nature of the conditional posterior distribution of $\boldsymbol{\beta}$ (name of density, explicit expression of mean, explicit expression of variance-covariance matrix).

Part (b), 6 points

Similarly, for the conditional posterior of error variance σ^2, given $\boldsymbol{\beta}$, we obtained:

$$\begin{aligned} &p\left(\sigma^2|\boldsymbol{\beta}, \mathbf{y}, \mathbf{X}\right) \propto \\ &\left(\sigma^2\right)^{-(\nu_0+1)} exp\left(\frac{-\tau_0}{\sigma^2}\right) * \\ &\left(\sigma^2\right)^{-n/2} exp\left(-\frac{1}{2\sigma^2}\left(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\right)'\left(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\right)\right) \end{aligned} \tag{3}$$

a Manipulate this expression to obtain the form

$$p\left(\sigma^2|\boldsymbol{\beta}, \mathbf{y}, \mathbf{X}\right) \propto \left(\sigma^2\right)^{-(\nu_1+1)} exp\left(-\frac{\tau_1}{\sigma^2}\right) \tag{4}$$

where ν_1 and τ_1 are placeholders for more complex expressions. Show the explicit form of ν_1 and τ_1.

b Recognizing the expression in (4) as the kernel of an inverse-gamma density, draw conclusions as to the exact nature of the conditional posterior distribution of σ^2 (name of density, explicit expression of shape ν_1, explicit expression of scale τ_1).

Part (c), 6 points

a Explain, in words, the basic mechanics of a *Gibbs Sampler* that utilizes these conditional distributions to obtain draws from the (unconditional) joint posterior.

b List four common diagnostics tools available to assess simulation noise, convergence of the chain of draws, as well as autocorrelation between draws. *(A verbal response is sufficient).*

Macroeconomics
Qualifying Exam Re-take
Fall 2019

Consider the Solow growth model with a Cobb-Douglas production function. Time t output is $Y_t = B(K_t)^{\alpha}(L_t)^{1-\alpha}$; K_t is the time t capital stock and L_t is time t labor. The labor input grows at the exponential rate n, $L_t = L_0 \exp(tn)$.

The capital accumulation equation is

$$\partial K_t / \partial t = sB(K_t)^{\alpha}(L_t)^{1-\alpha} - \delta K_t .$$

(a) Let $k_t \equiv K_t / L_t$. Find the equation describing the rate of change in k_t, $\partial k_t / \partial t$.

(b) Find the steady state values for k_t, the real wage, and the marginal product of capital.

(c) Initially, time 0, the economy is in its steady state. Suppose at time 0 there is a permanent increase in B, total factor productivity. Find the time path of k_t, the real wage, the marginal product of capital and labor's share of output as the economy converges to its new steady state.

Introduce technological change into the model in the following way:

$$Y_t = K_t^{\alpha}[E_t L_t]^{1-\alpha}, \text{ in which } [\partial E_t / \partial t]/E_t \equiv g, \text{ a constant.}$$

The capital accumulation equation is

$$\partial K_t / \partial t = s(K_t)^{\alpha}(E_t L_t)^{1-\alpha} - \delta K_t .$$

Consider the following definitions:

$$\hat{k}_t \equiv K_t / E_t L_t \equiv K_t / \hat{L}_t \text{; and}$$

$$\hat{y}_t \equiv Y_t / E_t L_t \equiv Y_t / \hat{L}_t .$$

(d) Find the equation describing the rate of change in $\hat{k}_t$, $\partial \hat{k}_t / \partial t$.

(e) Find steady state value for $\hat{k}_t$ and $\hat{y}_t$.

(f) Determine and graph the steady state time paths for k, the real wage, the marginal product of capital and labor's share of output.