Microeconomics Qualifying Exam, August 2020

Problem 1 (40 minutes)

Suppose $X = \mathbb{R}^2_+$ and $\mathscr{A} = \{\mathbb{B}(\mathbf{p}, m) | \mathbf{p} = (p_1, p_2) \gg 0 \text{ and } m > 0\}$.

a) Write down the definition of WARP for choice functions $C : \mathscr{A} \to X$.

b) Let $C : \mathscr{A} \to X$ be a choice function such that

$$C(\mathbb{B}(\mathbf{p}, m)) = \Big(\frac{m}{\sqrt{p_1}(\sqrt{p_1} + \sqrt{p_2})}, \frac{m}{\sqrt{p_2}(\sqrt{p_1} + \sqrt{p_2})}\Big).$$

Show that C satisfies WARP.

c) Let $C : \mathscr{A} \to X$ be a choice function such that

$$C(\mathbb{B}(\mathbf{p}, m)) = \Big(\frac{m}{p_1(1 + p_2)}, \frac{m}{1 + p_2}\Big).$$

Show that C violates WARP.

Problem 2 (50 minutes)

There are four players in the following participation game. Each player $i \in N = \{1, 2, 3, 4\}$ is deciding whether to participate in a production of a public good. Each player's participation cost is $c = \frac{1}{4}$. At least two players are needed in order to successfully produce the public good. Each player receives payoff 1 when the public good is successfully produced, but receives payoff 0 otherwise.

a) Describe the above participation game as a normal-form game.

b) Find all pure strategy NE of this game.

c) Find all symmetric mixed strategy NE of this game.

d) Find all NE of this game in which Players 1 and 2 play pure strategies and Players 3 and 4 play mixed strategies.

Problem 3 (40 minutes)

(a) Consider first a firm that uses a single input (factor) and produces a single output by means of the production function

$$f(x) = A{\cdot}\ln(1+x) \text{ for } x \geq 0$$

where $A > 0$ is a constant.
SOLVE the profit maximization problem

$$\max\ p{\cdot}f(x) - wx \text{ s.t. } x \geq 0$$

where $p > 0$ is the output price and $w > 0$ is the factor price.

(b) Consider next a firm that operates $K \geq 2$ factories $k = 1, \ldots, K$. In each factory, it produces the same product and uses the same factor of production.
Factory k has the production function

$$f_k(x) = A_k{\cdot}\ln(1+x) \text{ for } x \geq 0$$

where $A_k > 0$ is a constant and $A_1 > A_2 > \ldots > A_K$.
Suppose the output price is $p > 0$, the factor price is $w > 0$, and the firm is maximizing profit.
How much output is the firm producing and how much input is it using? You may use the result under (a) and what you know about aggregation.

Problem 4 (50 minutes)

Consider a pure exchange economy $\mathcal{E} = \{(X^i, \succsim^i, \omega^i)_{i=1}^I\}$ with two consumers $i \in \{1, 2\}$, two commodities $l \in \{1, 2\}$ and the initial endowments: $\omega^1 = (0, 10)$ and $\omega^2 = (10, 0)$.

Consumer i chooses among commodity bundles in $X^i = \mathbb{R}^2_+$ according to preferences $\succsim^i$ represented by the following utility function:

$$u^1(x_1^1, x_2^1) = x_1^1 + \ln x_2^1 \qquad \text{and} \qquad u^2(x_1^2, x_2^2) = x_1^2 + \ln x_2^2.$$

Let $p \in \mathbb{R}^2_+$, $p \neq 0$, denote a price vector.

(a) Draw an Edgworth box and determine formally the set of Pareto optimal allocations.

(b) Determine i's demand correspondence $x^i(p, \omega^i)$.

(c) Is the demand correspondence $x^i(p, \omega^i)$ homogeneous of degree 0?

(d) Determine the Walrasian equilibrium for the exchange economy $\mathcal{E}$.

(e) For each Pareto optimal allocation, determine the price equilibrium with transfers that generates the allocation. That is, determine the price vector p, the distribution of incomes m^1 and m^2, and the transfers T^1 and T^2 such that $T_1 + T_2 = 0$.

Econometrics Qualifying Exam, August 2020

Question 1 (suggested time: 20 min.)

1. **(a)** State and explain the Gauss-Markov theorem in the context of the Linear Regression model, assuming that $\sum_{t=1}^{n}(x_t - \overline{x})^2 \neq 0$, and some of the probabilistic assumptions in table 1, hold.

Table 1: Traditional Linear Regression model

$$Y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \; t=1,2,...,n$$

$$\left.\begin{array}{l} (1)\; (\varepsilon_t | X_t = x_t) \backsim \mathsf{N}(.,.), \\ (2)\; E(\varepsilon_t | X_t = x_t) = 0, \\ (3)\; Var(\varepsilon_t | X_t = x_t) = \sigma^2, \\ (4)\; E(\varepsilon_t \varepsilon_s | X_t = x_t) = 0, \; t \neq s, \end{array}\right\} t, s = 1, 2, ..., n$$

(b) Compare and contrast the specification in table 1 with that of table 2 in terms of

(i) their assumptions (1)-(4) vs. [1]-[5] and how they differ, and

(ii) potential assement of their validity using a preliminary data analysis.

Table 2: Normal, Linear Regression Model

Statistical GM:	$Y_t = \beta_0 + \beta_1 x_t + u_t, \quad t \in \mathbb{N} := (1, 2, ..., n, ...)$	
[1] Normality:	$(Y_t \| X_t = x_t) \backsim \mathsf{N}(.,.), \; f(y_t \| x_t; \boldsymbol{\theta}) = \sigma\sqrt{2\pi}\exp\{-\frac{(y_t - \beta_0 - \beta_1 x_t)^2}{2\sigma^2}$	$t \in \mathbb{N}$.
[2] Linearity:	$E(Y_t \| X_t = x_t)) = \beta_0 + \beta_1 x_t,$	
[3] Homoskedasticity:	$Var(Y_t \| X_t = x_t) = \sigma^2,$	
[4] Independence:	$\{(Y_t \| X_t = x_t), \; t \in \mathbb{N}\}$ independent process,	
[5] t-invariance:	$\boldsymbol{\theta} := (\beta_0, \beta_1, \sigma^2)$ are *constant* over t,	

$$\beta_0 = [E(Y_t) - \beta_1 E(X_t)] \in \mathbb{R}, \; \beta_1 = \frac{Cov(Y_t, X_t)}{Var(X_t)} \in \mathbb{R}, \; \sigma^2 = (Var(Y_t) - \frac{[Cov(Y_t, X_t)]2}{Var(X_t)}) \in \mathbb{R}_+,$$

(c) (i) Explain why the formulae for the OLS estimators of (β_0, β_1) coincide with those of the Maximum Likelihood (ML) estimators. (ii) Despite that, "the OLS [under (2)-(4)] and ML [under [1]–[5]] estimators of (β_0, β_1) have different finite sampling distributions and optimal *finite* sample properties". Explain your answer.

(d) Discuss the limitations of the Gauss-Markov theorem for inference purposes and explain why its results are not informative enough to test the hypotheses: H_0: $\beta_1 = 0$ vs. H_1: $\beta_1 \neq 0$.

Question 2 (suggested time: 20 minutes)

The following figures show a comparison of expected annual salaries for foreign ("F") and domestic ("D") players as a function of the number of years of playing experience for four different sports leagues.

You can think of these lines as regression lines of the general form $E(y|x, F, D)$, where y are earnings, x are years of experience, and F and D are binary indicators for foreign or domestic status. ($E(.)$ is the expectation operator)

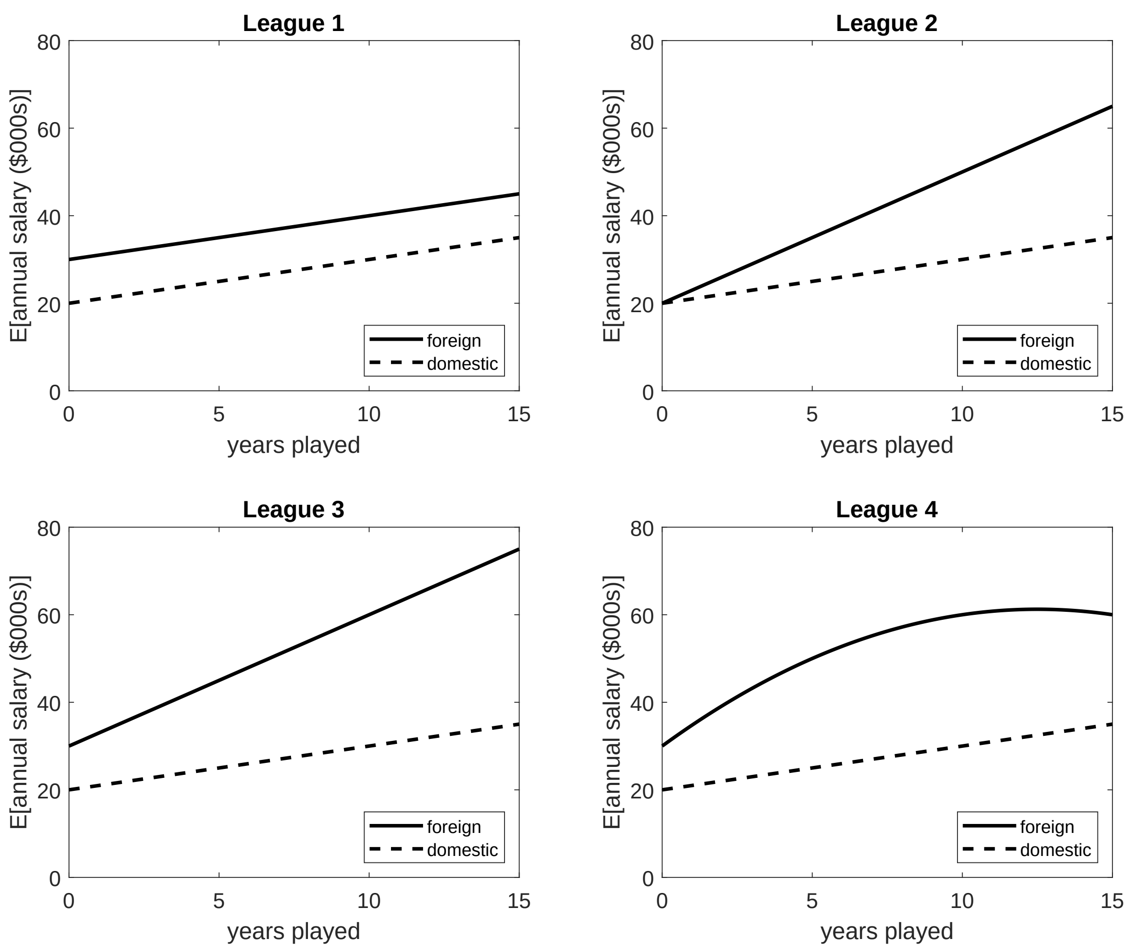

Figure 1: Expected player salaries in four different sports leagues

Part (a)

Consider the first league ("League 1"). Assume you want to characterize the salary model *with a single linear regression*. For this and all following parts of this question you can assume there are no other confounding variables that drive salaries, and that the regression error has the usual "CLRM" properties.

1. Show the *two* correct options for this regression model if the regression has a common intercept, for a single player (you can index all variables by "i"). Your choice of how you label parameters, as long as it is clear.

2. Now write down an equivalently correct model that does not have a common intercept.

Part (b)
Now consider "League 2." As before, your objective is to design a single linear regression that correctly captures the relationships shown in the figure for that league. Show *both* options to write the model.

Part (c)
Now consider "League 3." As before, your objective is to design a single linear regression that correctly captures the relationships shown in the figure for that league.

1. Show *both* options to write the model *without a common intercept.*

2. Show *all four* options to write the model *with a common intercept*

Part (d)
Now consider "League 4." As before, your objective is to design a single linear regression that correctly captures the relationships shown in the figure for that league. Write the model with a *common intercept*, using the "D" category as *implicit baseline.*

Part (e)
Consider again the "League 2" scenario from part (b). Specifically, consider the model that explicitly uses the "F" indicator. Assume you have estimated all coefficients for this model. Using your notation (and adding a "hat" symbol to all coefficients), what is the marginal effect of an additional year of playing on the annual salary of a domestic and foreign player, respectively?

Continuing with this model: In simple terms, and assuming you know all (estimated) variances and covariances for your original estimated coefficients (choose your own notation), show the expression for the standard error of the marginal effect of years played on annual salary for a *foreign* player.

Part (f)
Of the four leagues considered, in which case(s) could you have estimated the salary / years played relationship with *separate regressions for each group* without loss of efficiency (other than through ignoring the common variance)? Explain (A verbal response is sufficient).

Question 3 (suggested time: 20 minutes)

The Registrar at VT sampled at random students who had received at least one course grade of F at the end of their freshman year. Let y_i be the observed number of F's for sampled student i.

For the sample described, note that $\sum_{i=1}^{n} y_i = 38$, $\sum_{i=1}^{n} ln\left(y_i\right) = 9.76$, and $n = 20$.

You first assume that the distribution of y_i can be characterized by the (continuous) Pareto distribution, given as:

$$\begin{aligned} f\left(y_i\right) &= \theta y_i^{-\theta-1} \\ F\left(y_i\right) &= 1 - y_i^{-\theta} \quad \text{with} \quad y_i \geq 1;\ \theta > 0 \end{aligned} \tag{1}$$

where $f\left(.\right)$ and $F\left(.\right)$ denote, respectively the *pdf* and *cdf* for this distribution.

Part (a)

1. Write the log-likelihood for the i^{th} observation and the full sample.
2. Derive the gradient for the sample and solve (analytically and numerically) for the MLE estimator of θ (call it $\hat{\theta}$). Report the numerical result to a precision of *four decimals.*
3. Does the second order condition confirm that your estimator of θ does indeed maximize the likelihood? Explain.
4. Given your results, what is the estimated probability that a student in this population fails more then two courses, i.e. that $y_i > 2$? (Hint: Recall that the *cdf* for the Pareto density is given above). Report this numerical result to a precision of *four decimals.*

Part (b)

Now consider the *truncated-at-one Poisson* as an alternative sample distribution for y_i. Recall that the *un*-truncated Poisson density is given as:

$$f\left(y_i\right) = \frac{exp\left(-\lambda\right)\lambda^{y_i}}{y_i!}, \quad y_i = 0, 1, 2\ldots;\ \lambda > 0$$

The truncated-at-one Poisson must satisfy $y_i = 1, 2, 3\ldots$, so it excludes the value of 0. Note that, given that the Poisson is a *discrete* density, the truncated Poisson can be generically expressed as:

$$f\left(y_i | y_i > 0\right) = \frac{f\left(y_i\right)}{Pr\left(y_i > 0\right)} = \frac{f\left(y_i\right)}{1 - f\left(0\right)}$$

1. Derive the explicit form of the density for the truncated-at-one Poisson. Simplify as much as possible, such that only λ^{y_i} remains in the numerator of the resulting fraction.
2. Write the log-likelihood for the *ith* observation and the full sample.

3. Suppose that the maximum likelihood estimator of λ for this sample is 1.46. What is the estimated probability that a student in this population fails more than two courses?

4. *In words*, how would you decide on which model (Pareto or truncated Poisson) better fits the data: (a) In a classical estimation framework, using predictive measures of fit (list at least 3 examples) (b) in a Bayesian estimation framework (assuming any reasonable prior for θ and λ, respectively)?

[1] 30 minutes
Some argue that a consumption tax will decrease consumption and increase saving. Is this true? To answer consider the following two-period utility maximization problem. Explain your answers.

The government taxes consumption at the rate τ and gives each individual a lump-sum payment at time *t* equal to g_t. Each individual takes the lump-sum payment as *being fixed independently* of his own actions, but in fact, in equilibrium $g_t = \tau c_t$

Individuals are endowed with *e* units of output in the first period of life and wish to consume in the first and second period. Utility is

$$(c_1)^{1-\gamma}/(1-\gamma)+(c_2)^{1-\gamma}/(1-\gamma).$$

The gross return from saving is (1+*r*). The budget constraint is

$$(1+\tau)c_1+(1+\tau)c_2/(1+r)=e+g_1+g_2/(1+r).$$

(a) Solve the individual's utility optimization problem by way of determining desired time 1 and time 2 consumption.

(b) Time 1 saving is $e+g_1-(1+\tau)c_1$. Show saving is independent of the tax rate. Explain the intuition for this result.

[2] 30 minutes
Some years ago James Tobin (1981 Nobel Laureate) argued that the effects of an increase in the money supply depend upon the means by which the increase is brought about.

Use the Fiscal Theory of the Price Level model, as described in equations 1 and 2, to illustrate Tobin's point. Do this by considering two ways of increasing the money supply and contrast the effects on P_0. (This version of the FTPL is a model of P_0 and R.) Explain your answers.

***FE* :** $$\frac{B_0^g + M_0}{P_0} = S_0^f + S_0^s(R) \text{ and} \qquad \textbf{[1]}$$

***ME*:** $$\frac{M_0}{P_0} = \phi(R)\,. \qquad \textbf{[2]}$$