Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, August 21, 2017

Part 1: Microeconomics

3 Questions, 2 pages

Note: The minutes assigned to each question indicate the weight given to the question. For example, question 2 is 50 minutes out of a total of 180 minutes and thus counts for 5/18 of the grade for this exam.

Problem 1 (60 min.)
An owner is considering a first-price, sealed-bid auction for the sale of his property. There are two bidders whose valuations of the property, v_1 and v_2, are independent and identically distributed over $[0, 1]$. Assume that the valuation of every bidder is less than x with probability $F(x) = x^2$, for any $x \in [0, 1]$. Each buyer knows the existence of the other bidder, but the realized valuations are private information. The seller and the two buyers all know the cumulative distribution function F.

(a) Find the symmetric Bayesian Nash equilibrium (BNE).
HINT. Look for a BNE where the players use the same increasing linear strategy: $b_i(v_i) = \alpha v_i$, for any $i \in \{1, 2\}$ and $v_i \in [0, 1]$. You have to find the equilibrium value of α.

(b) Compute the expected revenue of the seller under the BNE of this first-price auction.
HINT. For each bidder i, determine i's expected payment given v_i. Then determine i's unconditional expected payment by integrating with respect to v_i.

(c) Suppose that the seller instead decides to use a second-price, sealed-bid auction.

(c.1) Argue that it is a weakly dominant strategy for each buyer to bid their exact valuation. What is then the BNE of this second-price auction?

(c.2) Compute the seller's expected revenue. Is it higher than under the first-price auction?

Problem 2 (50 min.)
Consider a monopolist who produces output $q \geq 0$ using two inputs, with quantities $x_1 \geq 0$ and $x_2 \geq 0$ and production function $f(x_1, x_2) = x_1 x_2$. The monopolist also produces its own inputs, at costs $C_1(x_1) = 30x_1$ and $C_2(x_2) = 10x_2^3$.

(a) Determine $C(q)$, the minimum cost of producing output $q \geq 0$.

(b) Suppose the monopolist faces the inverse market demand $P(q) = 160 - 128q^{1/4}$ for $q^{1/4} \leq \frac{160}{128} = \frac{5}{4}$ and $P(q) = 0$ for $q^{1/4} > \frac{5}{4}$. Determine the monopolist's profit maximizing output.

HINT. It proves convenient to express profits in terms of $z = q^{1/4}$.

Problem 3 (70 min.)
A pure exchange economy $\mathcal{E} = \{(X^i, \succsim^i, \omega^i)_{i=1}^I\}$ with three consumers $i \in \{1, 2, 3\}$ and two commodities $l \in \{1, 2\}$ is considered. The initial endowments are given by $\omega^1 = (4, 4)$, $\omega^2 = (4, 4)$, and $\omega^3 = (2, 2)$. A consumer i chooses among commodity bundles in $X^i = \mathbb{R}_+^2$ according to i's preferences $\succsim^i$ represented by the respective utility functions:

$$u^1(x_1^1, x_2^1) = x_1^1, \quad u^2(x_1^2, x_2^2) = x_1^2, \quad \text{and} \quad u^3(x_1^3, x_2^3) = x_2^3.$$

Let $p \in \mathbb{R}_+^2$, $p \neq 0$, be a price vector.

(a) Determine the set of all core allocations.

(b) Provide the definition of Walrasian equilibrium (as exact as possible).

(c) Determine the Walrasian equilibrium for the exchange economy $\mathcal{E}$.

(d) Determine the core allocations that can be supported by a price equilibrium with transfers (PET). For this, determine the price vector p, the distribution of wealths m^1 and m^2, and transfers T^1 and T^2.

(e) Consider a second replica of $\mathcal{E}$, i.e., the replica economy $\mathcal{E}^2$. Does the Equal Treatment Property hold true? (Prove or disprove it).

Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, August 24, 2017

Part 2: Econometrics and Macroeconomics

Part 2A: Econometrics (3 Questions on 3 pages)
Part 2B: Macroeconomics (1 Question on 1 pages)

Note: Econometrics counts for 60% of this exam (with each question being weighted equally) and macro for 40%.

Question 1 (20 minutes)

(a) Consider the simple Normal model:

$$X_t \backsim \text{NIID}(\mu, \sigma^2),\ t{=}1, 2, ..., n, ...,$$

with σ^2 is known; 'NIID' stands for 'Normal, Independent and Identically Distributed'.

(i) State the sampling distributions of the test statistic:

$$d(\mathbf{X}){=}\frac{\sqrt{n}(\overline{X}_n-\mu_0)}{\sigma},\quad \overline{X}_n{=}\frac{1}{n}\sum_{t=1}^{n} X_t,$$

under the null and under the alternative hypotheses:

$$H_0\!:\ \mu=\mu_0,\ \text{vs.}\quad H_1\!:\ \mu>\mu_0. \tag{1}$$

(ii) Define the optimal Neyman-Pearson (N-P) test in terms of $d(\mathbf{X})$ and explain what 'optimal' means in this case.

(b) (i) Compare and contrast the sampling distributions of $d(\mathbf{X})$ under H_0 and H_1 to:

$$d(\mathbf{X};\mu)=\frac{\sqrt{n}(\overline{X}_n-\mu^*)}{\sigma} \overset{\mu=\mu^*}{\backsim} \mathsf{N}(0,1), \tag{2}$$

where μ^* denotes the 'true' value of μ, whatever that happens to be.

(ii) Explain how (2) can be used to construct an $(1-\alpha)$ Confidence Interval for μ.

(c) Define and explain the notions of (i) type I error probability, (ii) type II error probability, (iii) power of the test and (iv) the p-value, and (v) compare and contrast (i) and (iv).

(d) (i) State the fallacies of acceptance and rejection and explain why the accept/reject rules and the p-value are vulnerable to these fallacies when they are interpreted as providing evidence for or against the null or the alternative.

(ii) Relate your answer in (i) to the difference between statistical and substantive significance.

Question 2

Suggested time: 20 minutes

Consider the true CLRM, given at the observation level as:

$$\begin{aligned} y_i &= \alpha + \mathbf{x}_i'\boldsymbol{\beta} + \epsilon_i, \quad \text{with} \\ \epsilon_i &\sim n\left(0, \sigma_\epsilon^2\right) \quad \forall i, \end{aligned} \tag{1}$$

where $\mathbf{x}_i$ is a vector of explanatory variables, and the individual error term ϵ_i are, as usual, uncorrelated across observations and with any of the $\mathbf{x}_i$'s.

Assume that the dependent variable y_i is measured with error ν_i, such that

$$\begin{aligned} \tilde{y}_i &= y_i + \nu_i, \quad \text{with} \\ \nu_i &\sim n\left(0, \sigma_\nu^2\right) \quad \forall i, \end{aligned} \tag{2}$$

where $\tilde{y}_i$ is the actually *observed* dependent variable used in the analysis, and n denotes the normal density. Further assume that ν_i is uncorrelated with $\mathbf{x}_i$ and ϵ_i for all observations.

Part (a)

(a) Write the true model in (1) at the sample level (you can call the data matrix $\mathbf{X}$). Then write the observational model, which uses $\tilde{\mathbf{y}}$ instead of $\mathbf{y}$, at the full-sample level.

(b) At the sample level, show the *true form* of the error vector for the observational model, as well as its expectation and variance. Call this error vector $\boldsymbol{\omega}$.

(c) Using partitioned regression, show that this model, despite the mis-measured dependent variable, produces unbiased estimates for both the intercept α and the coefficient vector $\boldsymbol{\beta}$.

(d) In summary, what is the only disadvantage for estimation introduced by this type of measurement problem?

Part (b)

Now assume, instead, that the measurement error in (2) takes the following form:

$$\nu_i \sim n\left(\mu_\nu, \sigma_\nu^2\right), \quad \mu_\nu \neq 0 \tag{3}$$

(a) Show the expectation and variance for $\boldsymbol{\omega}$ in this case.

(b) Show that this model produces an unbiased estimate for the coefficient vector $\boldsymbol{\beta}$, but a biased estimate for the intercept.

(c) Building on your results from above, explain what would happen if, instead, we had $\nu_i \sim n\left(\mu_i, \sigma_\nu^2\right)$.

Question 3

Suggested time: 20 minutes

Consider a city located downstream of a river dam. To facilitate spawning runs of salmon, the government decides to remove the dam. This will place a large fraction of residential properties in the city in a "Special Flood Hazard Area" (SFHA) with a higher risk of flooding during storm events. You are interested in estimating the loss in property values from being located in a SFHA. You collect data on the sale price of each home i, y_i, as well as many structural and neighborhood characteristics $\mathbf{x}_i$. Your plan is to run the following CLRM:

$$y_i = \alpha + \mathbf{x}_i'\boldsymbol{\beta} + \gamma d_i + \epsilon_i, \tag{1}$$

where d_i is a binary indicator that takes the value of 1 if a residence is located in a SFHA, and a value of zero otherwise.

Part (a)

(a) Write the model at the sample level, using notation $\mathbf{y}$, $\mathbf{X}$, $\mathbf{d}$, and $\boldsymbol{\epsilon}$.

(b) Assume the functional form of the regression equation is correct. Under which conditions, involving $\mathbf{X}$, $\boldsymbol{\epsilon}$, and $\mathbf{d}$ will the OLS solution $\hat{\gamma}$ be an unbiased estimate of the true SFHA effect?

(c) Now assume you have an omitted variable problem, that is a correlation of $\boldsymbol{\epsilon}$ with one or more elements of $\mathbf{X}$. Under which condition will remain $\hat{\gamma}$ be an unbiased estimate of the true SFHA effect? How would *pre-matching the sample* before running this regression help in this case? Explain in detail.

Part (b)

Now assume that $\mathbf{x}_i$ includes all relevant control variables, such that there are no O.V. problems. ***In words***, and using as much detail as necessary, describe how you would estimate the true SFHA effect using the following alternative methods. For each case, be specific in how you would estimate the Average Treatment Effect on the Treated (ATT), and its standard error.

(a) A separate regression approach - one regression using only the treated observations, and a second regression using only controls. When would that be useful or warranted?

(b) A regression model such as (1), but using the propensity score instead of $\mathbf{x}_i$. Under which conditions involving the PS would this produce a consistent estimate of the SFHA effect?

(c) Using a matching estimator with regression correction. You can assume 1 nearest neighbor. What is the main advantage of this hybrid model over the other approaches?

Macroeconomics (1 hour)

Consider a two period model in which the economy is composed of *identical* agents. Each agent's lifetime utility is a function of time 1 and time 2 consumption. The function is $U(c_1, c_2) = \ln c_1 + (\ln c_2)/(1+\theta)$.

Time t output of each agent is y_t. Output is non-storable. There is a time 1 loans market. The interest rate is r. The demand for bonds is b (which can be positive or negative). The budget constraints are

Time 1: $y_1 - c_1 - b = 0$. Time 2: $y_2 + b(1+r) - c_2 = 0$.

(a) An agent maximizes utility subject to the two budget constraints. Find the optimal values for c_1 and b as functions of r.

(b) Find equilibrium values for r, c_1, b, and c_2. Explain your answers.

(c) Under what circumstances will the equilibrium interest rate be less than or equal to zero. Explain the intuition behind your answer.

In contrast to our previous assumptions, assume output is storable, although it may deteriorate some. Let i be the inventory of goods carried over into time 2. The gross return to holding inventories is $1-\rho$, the real rate of return is $-\rho$, $0 \leq \rho < 1$.

The budget constraints are
Time 1: $y_1 - c_1 - b - i = 0$. Time 2: $y_2 + b(1+r) + (1-\rho)i - c_2 = 0$, $i \geq 0$.

Bonds and inventories are perfect substitutes. This is to say that if the two assets have the same returns, an individual is indifferent between holding one or the other.

(d) Given r, an agent selects c_1, c_2, b and i to maximize utility. Write the Lagrangian for this problem and determine the necessary first order conditions for the optimization problem. (Be sure to take account of the inequality constraint, $i \geq 0$.)

(e) If in equilibrium $i>0$, what is the equilibrium real interest rate?

(f) Find equilibrium values for c_1, b, i and r. (Hint: You must case this out. The answer is one thing for certain values of y_1 and y_2 and another for different values of y_1 and y_2.