MICRO QUALIFYING EXAM PROPOSALS, JAN 2015

Problem 1, Adam, 50 min.

Consider a private ownership economy with one consumer, called Robinson, one firm and two commodities. The first commodity is a consumption good. The second commodity is leisure; the difference between the total amount of time that Robinson has, 24 hours, and his working time l. Robinson is endowed with zero units of the consumption good. His preferences $\succsim$ on the consumption space $X = \{(x, l) \in \mathbb{R}^2_+ : 0 \leq l \leq 24, 0 \leq x\}$ are represented by the Cobb-Douglas utility function:

$$u(x, 24 - l) = x \cdot (24 - l).$$

The firm uses labor to produce the consumption good. The firm has the production function:

$$f(z) = \begin{cases} 0 & \text{if } 0 \leq l \leq 3; \\ 2\sqrt{l-3} & \text{if } l > 3. \end{cases}$$

The perfectly competitive firm is owned by Robinson. Denote by p the price of the consumption good and by w the wage rate, with $(p, w) \in \mathbb{R}^2_{++}$.

1) Derive Robinson's demand function $x^R(p, w)$ and supply function $l^R(p, w)$.

2) Draw the technology in a diagram. What returns to scale does the technology display?

3) Derive the firm's supply function $x^F(p, w)$ and its demand function $l^F(p, w)$.

4) Determine the Walrasian equilibrium, provided it exists.
   Use price normalization $w = 1$, if possible.

5) Formulate the Second Welfare Theorem. Does the theorem hold true in this economy? Please, provide an explanation.

Problem 2, Eric, 50 min.

Consider an infinitely repeated Bertrand oligopoly game with $n \geq 2$ firms. Each firm has the unit cost $c > 0$ and the discount factor $\delta \in (0,1)$. The market demand is given by $x(p)$, where x is a continuous function of the price. Let p^m be the monopoly price associated with the demand function $x(p)$; and consider the following Nash reversion strategy for firm i: "*play* $p_i = p^m$ *until any firm deviates, and play* $p_i = c$ *in every period after the first deviation*". Finally, let Π^m denote the monopoly profit, that is to say, $\Pi^m = (p^m - c)x(p^m)$.

(a) Determine the values of the discount factor δ for which the monopoly price can be sustained (in every single period) as the outcome of a subgame perfect equilibrium where all firms use the above Nash reversion strategy.

(b) Answer the same question for each of the variants below.

(b.1) At the end of each period, the market ceases to exist with probability $\alpha \in (0,1)$.

(b.2) The market demand is growing at rate $r > 0$; and therefore, in each period $t \in \mathbb{N}$, we have $x_t(p) = (1+r)^t x(p)$.

(b.3) It takes the firms L periods to (detect and) respond to a deviation, where $L \geq 2$.

Problem 3, Hans, 80 min.

Agland has an agrarian economy with two factors of production, land and labor, which are used to produce food. Each farm uses a constant returns to scale technology, given by the Cobb-Douglas production function $f : \mathbb{R}^2_+ \to \mathbb{R}$ with $f(k,l) = k^{1/2}l^{1/2}$ for $(k,l) \in \mathbb{R}^2_+$. That is, it produces $f(k,l)$ units of food when using k units of land and l units of labor as inputs.

PART (A)

- Solve the farm's Cost Minimization Problem (CMP)

$$\min\ p_k k + p_l l \quad \text{subject to} \quad f(k,l) = y$$

where $p_k > 0$ is the unit price (rental rate) of land, $p_l > 0$ is the unit price of labor (wage rate) and $y > 0$ is the desired farm output. You may use that at the solution of (CMP), the two conditions

$$\frac{MP_k}{MP_l} = \frac{p_k}{p_l} \qquad \text{tangency condition,} \tag{1}$$

$$f(k,l) = y \qquad \text{feasibility constraint} \tag{2}$$

have to hold where $MP_k = \dfrac{\partial f}{\partial k}$ and $MP_l = \dfrac{\partial f}{\partial l}$ are the marginal products. Denote the cost minimizing inputs by $k(p_k, p_l; y)$ and $l(p_k, p_l; y)$ and minimum cost by $c(p_k, p_l; y)$, that is $c(p_k, p_l; y) = p_k \cdot k(p_k, p_l; y) + p_l \cdot l(p_k, p_l; y)$.

PART (B)

From now on, assume that the unit price of food is 1.

- Using the cost function $c(p_k, p_l; y)$ obtained in PART (A), explain the following:

Every $y \geq 0$ is a profit maximizing output with zero profit (3)

if and only if

$$2(p_k p_l)^{1/2} = 1. \tag{4}$$

PART (C)

Suppose the population of Agland consists of $K > 0$ landowners and $L > 0$ workers. Each landowner owns one unit of land that is used for agricultural production. Each worker is endowed with one unit of labor that he supplies to the market. Landowners and workers are the only consumers and are only interested in food consumption.

- With prices $p_k > 0$ and $p_l > 0$, how much food does a landowner demand if she is not a worker?
- How much food does a worker demand if he is not a landowner?
- How much food would a consumer demand who is both a landowner and a worker?
- Determine X, the aggregate or market demand for food!

PART (D)
Now set

$$\begin{aligned} Y &= f(K,L) = K^{1/2}L^{1/2}, & (5)\\ &\quad \text{the aggregate output if all available inputs are used;} & \\ p_k &= \frac{1}{2}Y/K = \frac{1}{2}(L/K)^{1/2}; & (6)\\ p_l &= \frac{1}{2}Y/L = \frac{1}{2}(K/L)^{1/2}. & (7) \end{aligned}$$

(5) implicitly assumes that the markets for land and labor are cleared. It remains to clear the market for food.

- Show that the food market is cleared when factor prices are given by (6) and (7). You might proceed in three steps:

  First verify that (6) and (7) imply (4) so that (3) and (4) can be assumed, indeed, and Y is a profit maximizing output. (Because of the zero profit condition in (3), it does not matter how many farms there are and who operates them.)

  Second, use the findings from PART (A) to verify that $K = k(p_k, p_l; Y)$ and $L = l(p_k, p_l; Y)$.

  Third, show $Y = X$.

Part (E)
Suppose the economy of Ackerland is identical to that of Agland, except that a landowner in Ackerland owns two units of land. Assume that each consumer in both countries is either a landowner or a worker.

- Comparing equilibrium consumption, landowners in Ackerland consume more than landowners in Agland and workers in Ackerland also consume more than workers in Agland. Explain!
- Some workers might want to move from Agland to Ackerland. Which groups would benefit from such a migration? Which groups would be harmed by such a migration?
- When would the migration stop?

Econometrics

Question 1 (30 minutes):

1. **(a)** Explain what an estimator is and why its optimality can only be assessed via its sampling distribution.

(b) Explain briefly the following properties of an estimator:
(i) *weak consistency*, (ii) *strong consistency*, (iii) *unbiasedness, (iv) full efficiency.*
<u>Hint</u>: if it helps, illustrate your answers assuming a simple statistical model.

(c) In the context of the simple Bernoulli model:

$$X_t \backsim \text{BerIID}(\theta, \theta(1-\theta)),\ t{=}1,2,...,n,...,\ x_t{=}0,1,\ 0{\leq}\theta{\leq}1, \tag{1}$$

where 'BerIID' stands for 'Bernoulli, Independent and Identically Distributed',
discuss whether the following functions constitute possible estimators of θ :

(i) $\widehat{\theta}_1{=}X_n$, (ii) $\widehat{\theta}_2{=}\frac{1}{2}(X_1 - X_n)$,
(iii) $\widehat{\theta}_3{=}\frac{1}{3}(X_1{+}X_2{+}X_n)$, (iv) $\widehat{\theta}_n{=}\frac{1}{n}\sum_{i=1}^{n} X_i$,

(d) Using your answer in (c): (i) state which of the properties (i)-(iv) in (b) hold for the estimators in (c) and select the most optimal, explaining your answer.

(e) Using your answer in (d), explain why *relative efficiency* is practically useless when the 'best' estimator is inconsistent.

(f) Explain why the following definitions of:
(i) Unbiasedness:

$$E(\widehat{\theta}) = \theta,\ \text{for all}\ \theta{\in}\Theta, \tag{2}$$

and (ii) the Mean Square Error:

$$\mathsf{MSE}(\widehat{\theta}){=}E(\widehat{\theta}{-}\theta)^2{=}Var(\widehat{\theta}) + [Bias(\widehat{\theta})]^2, \quad \text{for all}\ \theta{\in}\Theta, \tag{3}$$

make no sense in the context of frequentist estimation because of the quantifier 'for all $\theta{\in}\Theta$'.

Question 2 (40 minutes)

Consider a simultaneous equation model based on $\underset{(N\times1)}{\boldsymbol{y_1}}, \underset{(N\times1)}{\boldsymbol{y_2}}, \underset{(N\times K)}{\boldsymbol{x_1}}, \underset{(N\times M)}{\boldsymbol{x_2}}$:

$$\begin{aligned} (1)\quad & \boldsymbol{y_1} = \gamma_1 \boldsymbol{y_2}^{\gamma_2} + \boldsymbol{x_1\delta_1} + \boldsymbol{u_1} \\ (2)\quad & \boldsymbol{y_2} = \gamma_3 \boldsymbol{y_1} + \boldsymbol{x_2\delta_2} + \boldsymbol{u_2} \end{aligned}$$

Furthermore, $E(\boldsymbol{u_1}|\boldsymbol{x_1}, \boldsymbol{x_2}) = E(\boldsymbol{u_2}|\boldsymbol{x_1}, \boldsymbol{x_2}) = 0$

a) [12 min] Suppose you are asked to estimate equation (2) only. How would you estimate this single equation <u>consistently and efficiently</u> using data on $\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{x_1}, \boldsymbol{x_2}$? Please explain your answer carefully and be clear about any assumptions you need to make.

b) [6 min] Suppose $\gamma_1 = 0$. Are $\boldsymbol{\delta_1}$ and γ_2 identified? Why or why not?

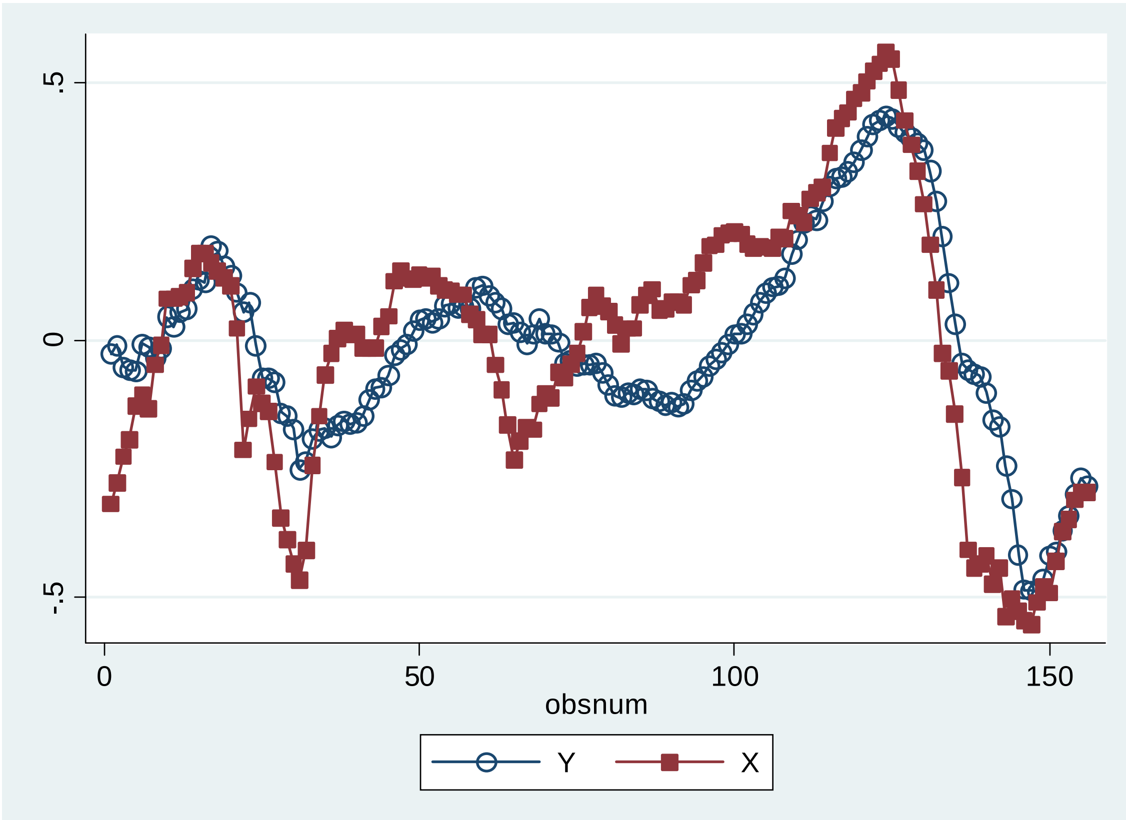
c) [14 min] Now suppose $\gamma_3 = 0$. Let $\boldsymbol{\hat{y}_2} = \boldsymbol{x_2\hat{\delta}_2}$, where $\hat{\delta}_2$ denotes the OLS estimates of equation (2) with the assumption of $\gamma_3 = 0$. Does Nonlinear Least Square (NLS) estimation of $\boldsymbol{y_1} = \gamma_1 \boldsymbol{\hat{y}_2}^{\gamma_2} + \boldsymbol{x_1\delta_1} + \boldsymbol{u_1}$ consistently estimate $\boldsymbol{\delta_1}$, γ_1, γ_2?

[Hint: there are multiple cases to consider as γ_1, γ_2 vary, you should present your answers by different cases where different values of γ_1, γ_2 are assumed: a. $\gamma_1 = 0$; b. $\gamma_1 \neq 0, \gamma_2 = 0$; c. $\gamma_1 \neq 0, \gamma_2 = 1$; d. $\gamma_1 \neq 0, \gamma_2 \neq 0 \textit{ or } 1$]

d) [8 min] Please write down the specific sample moment conditions you would use to simultaneously estimate all the parameters of the model in equation (1) and (2) using GMM. Note: Define $\boldsymbol{Z_i} = \boldsymbol{x_{i1}} \cup \boldsymbol{x_{i2}}$.

Question 3 (20 minutes)

Consider the 156 quarterly observations on Y_t and X_t plotted below against time. These data are observations on an actual U.S. macroeconomic time series from 1975 to 2013 and have been linearly detrended. Let ΔY_t be defined as the change in $Y_t \equiv Y_t - Y_{t-1}$.



a. Presuming that Y_t is the logarithm of Z_t, what (in a few words) does ΔY_t measure in relation to Z_t?

b. Letting $\tilde{Y}_t$ denote the original (trended) data, how was Y_t produced?

c. Do the two time series plotted above appear to exhibit a high degree of persistence? In other words, do the fluctuations in these time series tend to persist over time, lending a smooth appearance to the plotted time path of the series, or do the fluctuations instead tend to die out promptly? (Answer "yes" or "no.")

d. How would the persistence in Y_t be most simply quantified with a single sample statistic?

e. How could you quantify the persistence in Y_t with a simple linear regression equation?

f. Suppose that you estimated the parameter β in the regression model,

$$\Delta Y_t = \alpha + \beta Y_{t-1} + \lambda X_{t-1} + \gamma_1 \Delta Y_{t-1} + \ldots + \gamma_1 \Delta Y_{t-k} + U_t \qquad (1)$$

where X_{t-1} is some explanatory variable and *k* is chosen to be sufficiently large that serial correlation in U_t is apparently negligible.

i. Suppose that Y_t has a "unit root" – i.e., is I(1), or "integrated of order one," or needs to be differenced in order to have a well-defined (i.e., finite and constant) population variance, etc. Would you in that case expect the population value of β to be negative, zero, or positive?

ii. If Y_t is instead I(0), would you expect the population value of β to be negative, zero, or positive?

iii. Does the usual estimated t ratio on the OLS estimate of β yield a valid test of the null hypothesis that β is zero? {Hint: the answer to this question is "no." Under the null hypothesis that $Y_t \sim I(1)$, then the the sampling distribution of the estimated t ratio on $\hat{\beta}^{ols}$ is not Student's *t* but instead has a sampling distribution tabulated by Dickey and Fuller; consequently, this test is called the "augmented Dickey-Fuller" test or "ADF" test.}

iv. If the ADF test rejects its null hypothesis (at the 5% level, say) does this provide evidence that Y_t is so persistent as to be I(1)? Or does this result provide evidence that Y_t is not actually that persistent and that Y_t does not in fact have a unit root?

v. Unit root tests, such as the ADF test, are notoriously low in power – especially for highly persistent time series, where we might particularly need a good test. What then, does it mean if the ADF test fails to reject its null hypothesis?

Qualifying Macro Question (45 minutes)

The two parts are not related.

Part 1 (30 minutes): You learned in class how intermediate goods can be combined into a final good. It turns out that the same setup can be applied to consumption as well. Suppose there are only <u>three goods</u> in the economy and the consumer maximizes a *consumption index* C by combining the goods:

$$\max_{\{C_j\}} C \equiv \left[\sum_{j=1}^{3} C_j^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

subject to the budget constraint

$$\sum_{j=1}^{3} P_j C_j = PC$$

where the amount PC is exogenous.

a) Solve for the optimal consumption for each good in terms of C and the prices.

b) Define an appropriate *price index* P such that $C_j = \frac{1}{3}\left(\frac{P_j}{P}\right)^{-\theta} C$.

c) Suppose $P_1 = 1, P_2 = 2, P_3 = 1$ and that the total spending $PC = 1$. What is the consumption on each good (C_1, C_2, C_3) when $\theta = 2$? What happens when $\theta \to \infty$? Explain the intuition behind your results.

Part 2 (15 minutes): The economy is described by an AS curve and an AD curve:

$$y_t = \delta(p_t - E_{t-1}p_t), \qquad \delta > 0,$$

$$y_t = m_t - p_t.$$

If money supply is an AR(1) process $m_t = \rho m_{t-1} + \epsilon_t$ with $0 < \rho < 1$, is equilibrium output also an AR(1) process? Show your steps.

X 2

Macro Question 2 (45 minutes)

Consider an overlapping generations model of money. Each individual lives for two periods and is endowed with e_1 units of output in the first period of life and with e_2 units in the second period of life ($e_1 > e_2$). Output cannot be stored and the only way to save in the first period of life is to sell output to the old generation for money. Each generation is of equal size. Assusme without loss of generality this size in N=1. Employ the following notation:

(i) M= the fixed nominal supply of money (assume money is denominated as "dollars").

(ii) P_t = the dollar price of output at time t.

(iii) $R_t \equiv \frac{P_t}{P_{t+1}}$.

(iv) (c_{t1}, c_{t2}) is the consumption vector of the time t young person. c_{t1} is generation t's consumption in the first period of life and c_{t2} is t's consumption in the second period of life.

A time t young person seeks to maximize lifetime utility as given by

$$u(c_{t1}, c_{t2}) = \ln c_{t1} + \ln c_{t2}. \qquad [1]$$

Let m_t be the generation t individual's nominal demand for money. His time t and time t+1 budget constraints are

$$P_t e_1 = P_t c_{t1} + m_t \quad \text{and} \quad c_{t2} = e_2 + (m_t / P_{t+1}). \qquad [2]$$

(a) Reduce the two budget constraints in [2] to 1 lifetime budget constraint.

(b) For given values of P_t and P_{t+1} solve the generation t's utility maximization problem. Write c_{t1} and c_{t2} as functions of $R_t \equiv \frac{P_t}{P_{t+1}}$.

(c) Write $c_{t-1,2}$ as a function of $R_{t-1} \equiv \frac{P_{t-1}}{P_t}$

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In the following consider a stationary equilibrium in which $R_t = R$, a constant independent of time.

(d) At time t the output market must clear: $c_{t1} + c_{t-1,2} = e_1 + e_2$.
Use this equilibrium condition to determine the candidate stationary equilibria (there are two possible values for R.)

(e) One possible equilibrium is R=1. In this case the price level is a constant independent of time. Find the equilibrium price level in this case.

(f) For $R \neq 1$, find c_{t1}. What is the nominal demand for money, m_t, in this case? Is this a possible monetary equilibrium in which money has value?