Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, November 7, 2016

Part 1: Microeconomics

3 Questions, 4 pages

Note: The minutes assigned to each question indicate the weight given to the question. For example, question 1 is 50 minutes out of a total of 180 minutes and thus counts for 5/18 of the grade for this exam.

Problem 1 (50 min)

Consider an asymmetric Cournot oligopoly with $n > 1$ firms $i = 1, \ldots, n$. The linear inverse demand is

$$P(Q) = \begin{cases} a - Q \text{ if } Q < a; \\ 0 \text{ otherwise} \end{cases}$$

with $a > 0$. There are cost coefficients $0 < c_1 \leq c_2 \leq \ldots \leq c_n$. Firm i chooses output $q_i \geq 0$. It has cost $C_i(q_i) = \frac{1}{2}c_i q_i^2$ and profit

$$\pi_i(q_1, q_2, \ldots, q_n) = P(Q)q_i - C_i(q_i)$$

where $Q = \sum_{j=1}^{n} q_j$.

Your ultimate task is to find the Cournot equilibrium (Cournot-Nash equilibrium) $(q_1^*, q_2^*, \ldots, q_n^*)$.

(i) Argue that $Q^* = \sum_{j=1}^{n} q_j^*$ satisfies $0 < Q^* < a$.

(ii) Based on (i), argue that the First-Order Conditions (FOC) at the equilibrium hold with equality.

(iii) Write firm i's FOC in the form $q_i^* + \alpha_i Q^* = \beta_i$ where α_i and β_i depend on the parameters of the model.

(iv) Determine Q^* after summation of the n FOCs.

(v) Derive q_i^* for $i = 1, \ldots, n$.

(vi) Rank the equilibrium outputs q_i^*, $i = 1, \ldots, n$. Justify your answer!

Problem 2 (70 min)

Consider a private ownership economy with one consumer, called Robinson, one firm and two commodities. The first commodity is time which can be consumed as leisure or used as labor input in production. The second commodity is a consumption good.

The Consumer. Robinson consumes two goods, leisure with quantity x_1 and the consumption good with quantity x_2. Robinson is endowed with the positive amount $\omega_1 = 24$ of time and zero amount of the consumption good. That is, the endowment bundle is $\omega = (\omega_1, 0) = (24, 0)$.

Suppose that the price system assumes the form $p = (p_1, p_2) = (p_1, 1)$ which means that the consumption good serves as numéraire and p_1 is both the nominal and the real wage rate. If the firm makes zero profits, then the consumer's budget line is as in the below diagram. Suppose further that the tangency condition holds at the consumer's optimal consumption bundle (x_1^*, x_2^*), the solution of the problem

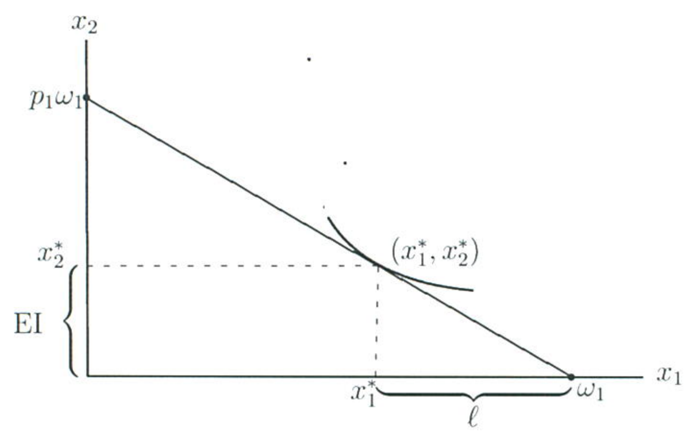
$$\max_{(x_1, x_2)} u(x_1, x_2) \text{ s.t. } px = p\omega + \pi(p),\ 0 \leq x_1 \leq 24,\ 0 \leq x_2 \tag{1}$$

where $\pi(P)$ is the firm's maximal profit and we assume that the budget constraint is binding. (x_1^*, x_2^*) is also shown in the diagram (for the case $\pi(p) = 0$). Moreover,

$\ell = \omega_1 - x_1^*$ is the consumer's labor supply;

$EI = p_1 \ell$ is the consumer's labor income or "earned income".

Notice that $EI = x_2^*$ because of $p_2 = 1$. **IMPORTANT: In general, EI is less than $p\omega = p_1\omega_1 + \pi(p)$, the income that enters the budget constraint!**



The Firm. The perfectly competitive firm is owned by Robinson. It uses labor to produce the consumption good. Its technology is given by the following production function $f : \mathbb{R}_+ \to \mathbb{R}_+$:

$$f(\ell) = \begin{cases} 0 & \text{for } 0 \leq \ell \leq 1; \\ 2\sqrt{\ell - 1} & \text{for } \ell > 1 \end{cases}$$

where $\ell \geq 0$ is the firm's labor input. The corresponding technology (production set) is

$$Y = \{(y_1, y_2) \in \mathbb{R}^2 \mid y_1 \leq 0, y_2 \leq f(-y_1)\}.$$

From now on we assume that Robinson has Cobb-Douglas preferences with utility representation $u(x_1, x_2) = x_1 x_2$ for $(x_1, x_2) \in X = \{(x_1, x_2) \in \mathbb{R}^2_+ \mid x_1 \leq 24\}$.

(a) Draw the firm's production function and technology in a diagram.

(b) What returns to scale does the technology display?

(c) Solve the firm's profit maximization problem

$$\max py \text{ s.t. } y \in Y$$

for each price vector $p = (p_1, 1)$ with $p_1 > 0$.

(d) Solve Robinson's utility maximization problem (1) for each price vector $p = (p_1, 1)$ with $p_1 \geq 0$.

(e) Determine the competitive or Walrasian equilibrium for this economy.

Problem 3 (60 min)

The inhabitants of Joyville can consume two commodities. Commodity 1 is quiche, with quantities $x_1 \geq 0$. Commodity 2 is strawberry ice-cream, with quantities $x_2 \geq 0$. Consider the following four types of consumers with respect to preferences.

- Type 1 exhibits perfect substitutes with utility representation $U(x_1, x_2) = x_1 + x_2$ for $(x_1, x_2) \in \mathbb{R}^2_+$.
- Type 2 cares only for quiche and has utility representation $U(x_1, x_2) = x_1$ for $(x_1, x_2) \in \mathbb{R}^2_+$.
- Type 3 likes quiche and dislikes strawberry ice-cream and has utility representation $U(x_1, x_2) = x_1 - x_2$ for $(x_1, x_2) \in \mathbb{R}^2_+$.
- Type 4 has lexicographic preferences: (x_1, x_2) is preferred to (y_1, y_2) if and only if $x_1 > y_1$ or $x_1 = y_1$, $x_2 > y_2$.

According to the census data, all inhabitants of Joyville consume only quiche. (Notice that in Joyville, the price of quiche is always slightly lower than the price of strawberry ice-cream.) Professor Meredith Smith has conducted an experiment with 100 students from Joyville College who were all born and raised in Joyville.† Each of the participants was given the choice between four consumption bundles: $A = (10, 6)$, $B = (10, 14)$, $C = (12, 10)$, $D = (12, 11)$. Each bundle provides sufficient nutrients for a two-day period. Each participant agreed to be monitored for two days and to consume nothing else than the selected bundle during those days. In the experiment, all possible choices A, B, C and D were observed.

(a) Do the census data rule out any of the four consumer types? Explain!

(b) Suppose a student chooses bundle A in the experiment. Which of the four consumer types can be ruled out for this student?

(c) Suppose a student chooses bundle B in the experiment. Which of the four consumer types can be ruled out for this student?

(d) Suppose a student chooses bundle C in the experiment. Which of the four consumer types can be ruled out for this student?

(e) Suppose a student chooses bundle D in the experiment. Which of the four consumer types can be ruled out for this student?

†Thanks to a generous donation, Professor Smith could pay a participation fee of \$500 per person.

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Part 2: Econometrics and Macroeconomics

Part 2A: Econometrics (3 Questions on 3 pages)
Part 2B: Macroeconomics (1 Question on 2 pages)

Macro Question (1 hour)

The policy maker's goal is to minimize the loss function:

$$Z_t = (U_t - kU_t^n)^2 + (\pi_t)^2 . \qquad [1]$$

U_t is the time t unemployment rate and U_t^n is the "natural" rate of unemployment. k is a positive constant less than 1.

The unemployment equation: $U_t = U_t^n - (\pi_t - \pi_t^e)$. [2]

π_t^e is expected inflation as determined by the public.

At the moment there is no uncertainty.

(a) Given expected inflation, π_t^e, the policy maker selects π_t to minimize [1] subject to [2]. What is the policy maker's optimal value for π_t as a function of π_t^e (the policy maker's reaction function)?

(b) Agents know the policy maker's objective function and thus know the policy maker's reaction function. Agents wish to minimize $(\pi_t - \pi_t^e)^2$. What is the optimal value for π_t^e?

(c) What are the equilibrium values for π_t and π_t^e as determined by the analysis in (a) and (b)?

(d) There is an excess of inflation in this model, an inflation bias. Explain. On what variable(s) does this bias depend? Explain.

(e) Introduce uncertainty into this model. Say, for example, the policy maker cannot control inflation precisely. Inflation is

$$\pi_t = \rho \pi_t * . \qquad [3]$$

ρ is a random variable with mean 1 and variance σ_ρ^2. The policy maker selects $\pi_t *$ to minimize the ***expected value*** of $(U_t - kU_t^n)^2 + (\pi_t)^2$, given (i) π_t^e, (ii) given knowledge of the distribution of ρ but not its exact value and (iii) given equations 2 and 3. What is the optimal value of $\pi_t *$?

(f) Agents know the policy maker's objective function and thus know the policy maker's reaction function in (e). Agents wish to minimize $(\pi_t - \pi_t^e)^2$. What is the optimal value for π_t^e?

(g) What are the equilibrium values for π_t * and π_t^e as determined by the analysis from in (e) and (f)?

(h) Explain why the answers in (g) and (c) differ.

Econometrics Question 1 (20 minutes)

1. Consider the Linear model as specified below:

$$Y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \;\; t{=}1,2,...,n,$$

$$\left.\begin{array}{l}(1)\ E(\varepsilon_t) = 0,\\ (2)\ Var(\varepsilon_t) = \sigma^2,\\ (3)\ E(\varepsilon_t\varepsilon_s){=}0,\end{array}\right\} t{\neq}s, \;\; t,s{=}1,...,n,$$
$$(4)\ \textstyle\sum_{t=1}^{n}(x_t{-}\overline{x})^2{\neq}0.$$

(a) State and explain the Gauss-Markov theorem.

(b) Derive the Maximum Likelihood Estimators (MLEs) of the parameters (β_0, β_1), by adding the Normality assumption:

$$(5)\ \varepsilon_t \backsim \mathsf{NIID}(0, \sigma^2), t{=}1,2,...,n.$$

Note that the density function of ε_t is: $f(\varepsilon_t){=}\frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{\varepsilon_t^2}{2\sigma^2})$.

(c) Assuming assumption (4) holds, compare and contrast the properties of the Least-Squares estimators as given by the Gauss-Markov theorem, with those of ML estimators in terms of their sampling distributions and their optimal finite sample properties.

(d) Discuss the limitations of the Gauss-Markov theorem and explain why its results do not provide enough information to test the hypotheses: H_0: $\beta_1{=}0$ vs. H_1: $\beta_1{\neq}0$.

Econometrics Question 2

20 minutes

You are interested in the fraction θ of all eligible VT students that participated in a recent election, with $0 \leq \theta \leq 1$. Your prior on θ is a Beta distribution, given as:

$$\begin{aligned} p(\theta) &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}, \quad \text{with} \\ \alpha,\beta &> 0, \\ E(\theta) &= \frac{\alpha}{\alpha+\beta} \\ V(\theta) &= E(\theta)\frac{\beta}{(\alpha+\beta)(\alpha+\beta+1)} \end{aligned} \tag{1}$$

You poll 100 randomly selected students, and 43 of them report that they did go and vote in the election.

You decide that the sample distribution ("likelihood function") of VT voters y, out of a sample of size n, is well characterized by a binomial density, given as:

$$p(y|\theta) = \binom{n}{y}\theta^{y}(1-\theta)^{n-y} \tag{2}$$

Part (a)

Based on past years of elections, the average participation rate was 0.2, with a variance of 0.1. Find prior parameters α and β to fit these moments.

Part (b)

Characterize the posterior kernel for θ and show that this is the kernel of another Beta density with parameters α_1, β_1. Show the explicit forms of these posterior parameters and derive their numerical values.

Part (c)

Derive the posterior expectation and variance of θ. How do these moments compare to your priors?

Econometrics Question 3

20 minutes

Consider the linear regression model, expressed for a single observation i:

$$y_i = \beta_1 + \beta_2 x_{2i}^* + \epsilon_i, \tag{1}$$

where ϵ_i has the usual CLRM properties.

Assume, however, that x_{2i}^* is measured with *proportional error* for the entire sample, with the relationship between the observed x_{2i} and the true x_{2i}^* given as:

$$x_{2i} = x_{2i}^* \left(1 + \alpha\right), \quad \text{with} \quad 0 < \alpha < 1 \tag{2}$$

Part (a)

Express the model in (1) in terms of x_{2i} for a single observation *and for the full sample.* Show that the measurement error can be interpreted as introducing omitted variable bias in a regression that uses $\mathbf{x}_2$ instead of $\mathbf{x}_2^*$.

Part (b)

Using partitioned regression, show that the estimated coefficient on $\mathbf{x}_2$ (call it b_2) is biased compared to the true β_2.

Part (c)

How could this problem be fixed if α were known?