Question 1. (a) State and explain [not just write down using symbols] the optimal properties of maximum likelihood estimators (finite sample and asymptotic), under the traditional regularity conditions; use a particular example if it helps (table 1).
(b) Compare and contrast the likelihood function and the distribution of the sample.
(c) For the simple (one parameter - σ^2 *known*) Normal model (table 1):

(i) derive the Maximum Likelihood Estimator (MLE) $\widehat{\mu}_{ML}$ of μ,

(ii) derive the sampling distribution of $\widehat{\mu}_{ML}$, including its mean and variance of $\widehat{\mu}_{ML}$, and explain how assumptions [1]-[4] play a crucial role in these derivations.

(d) Explain and comment on Le Cam's (1986) argument that:

"... limit theorems 'as n tends to infinity' are logically devoid of content about what happens at any particular n. All they can do is suggest certain approaches whose performance must then be checked on the case at hand. Unfortunately the approximation bounds we could get were too often too crude and cumbersome to be of any practical use." (p. xiv).

Table 1 - The simple (one parameter) **Normal model**		
Statistical GM:		$X_t = \mu + u_t,\ t\in\mathbb{N}$,
[1]	Normal:	$X_t \backsim \mathsf{N}(.,.)$,
	i.e. $f(x;\theta)=\frac{1}{\sigma\sqrt{2\pi}}\exp\{-\frac{(x-\mu)^2}{2\sigma^2}\},\ \mu\in\mathbb{R},\ x\in\mathbb{R}$	
[2]	Constant mean:	$E(X_t)=\mu$, for all $t\in\mathbb{N}$,
[3]	Constant variance:	$Var(X_t)=\sigma^2$ (known),
[4]	Independence:	$\{X_t,\ t\in\mathbb{N}\}$ - independent process.

Question 2

Suggested time: 20 min

Consider a CLRM of the following form, at the observation level (dropping individual-level subscripts for simplicity):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 x_2^2 + \epsilon, \quad \text{where} \\ \epsilon \sim i.i.d.\left(0, \sigma^2\right) \tag{1}$$

Part (a)

You are primarily interested in the marginal effect of x_1 and x_2 on the outcome variable, i.e. $\left(\frac{\partial y}{\partial x_1}\right)$, and $\left(\frac{\partial y}{\partial x_2}\right)$. Show the explicit form of these marginal effects, for a given x_1 and x_2.

Part (b)

Let $E\left(x_1\right) = \mu_1$ and $E\left(x_2\right) = \mu_2$ be the population means of the two explanatory variables. Similarly, let σ_1^2 and σ_2^2 be the two variances. Assume all of these moments are known to the analyst. Also, it is known that x_1 and x_2 are *independently* distributed.

Derive the expectation, over x_1 and x_2, of these marginal effects you obtained in the preceding part in terms of these moments. Let the solutions be labeled as γ_1 and γ_2, respectively.

Part (c)

Setting $x_2 = \mu_2$, at what value of x_1 is y maximized?
How do you know it's a maximum? (Assume $\beta_3 < 0$)

Part (d)

Using the results form part (b), solve for β_1 and β_2, then insert the resulting expressions into equation (1) in lieu of β_1 and β_2.

After some manipulation, this should produce the following "reduced-form" model:

$$y = \beta_0 + \gamma_1 x_1 + \gamma_2 x_2 + \beta_3\left(f\left(x_1, x_2, \mu_1, \mu_2, \sigma_1^2, \sigma_2^2\right)\right) + \epsilon, \tag{2}$$

where you need to fill in the explicit form of $f\left(.\right)$.

Part (e)

Now suppose that $\mu_1 = \mu_2 = 0$, and you estimate the model in (1). What is the interpretation of β_1 and β_2?

Question 3

Suggested time: 20 min

Consider the Poisson model for a random variate y with parameter λ, given as

$$\begin{aligned} p\left(y|\lambda\right) &= \frac{\lambda^{y} exp(-\lambda)}{y!}, \quad \text{with} \\ E\left(y|\lambda\right) &= V\left(y|\lambda\right) = \lambda, \quad \lambda > 0, y \in \{0, 1, 2, 3, ...\} \end{aligned} \tag{1}$$

Part (a)
Now consider a sample of n observations from this distribution, with each observation generically labeled $y_i, i = 1 \ldots n$. Write down the joint distribution for the sample data (in *un*-logged form). Call it $p\left(\mathbf{y}|\lambda\right)$.

Part (b)
Suppose you stipulate a *gamma* prior density for λ with shape parameter a and inverse scale ("rate") parameter b, given as

$$\begin{aligned} p\left(\lambda\right) &= g\left(a, b\right) = \frac{b^{a}}{\Gamma\left(a\right)} \lambda^{(a-1)} exp\left(-b\lambda\right), \quad \text{with} \\ E\left(\lambda\right) &= \frac{a}{b}, \; V\left(\lambda\right) = \frac{a}{b^{2}}, \quad \lambda, a, b > 0, \end{aligned} \tag{2}$$

Show that the posterior distribution of λ, given your collected data from the Poisson, is also a gamma. Show the form of the posterior shape and rate parameters (you can call them a^{*} and b^{*}).

Part (c)
Show that the posterior expectation can be written as a weighted average of the prior expectation and the sample mean. What happens to this posterior expectation as $n \to \infty$?

Part (d)
Suppose you are opening a small restaurant In Blacksburg. Before you start your business, you expect 20 guests / day with a variance of 10, which can be modeled as a gamma prior with shape 40 and rate 2. After 30 days of running your business, you count a total of 824 guests. You plot the daily counts, and they look exactly like a Poisson distribution.

How many guest *per day* would you expect for the following month?

Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, November 30, 2017

Part 2: Econometrics and Macroeconomics

Part 2A: Econometrics (3 Questions on 3 pages, 20 minutes each)
Part 2B: Macroeconomics (1 Question on 1 pages)

Note: Econometrics counts for 60% of this exam (with each question being weighted equally) and macro for 40%.

Macroeconomics (1 hour)

Firms are identical and output at each is $y_t = k_t^{\alpha} l_t^{1-\alpha}$. The government taxes the firm's output net of its wage bill at the rate τ. So a firm selects its desired capital stock and labor input so to maximize net profits as given by

$$(1-\tau)[k_t^{\alpha} l_t^{1-\alpha} - w_t l_t] - r_t k_t . \qquad [1]$$

In this equation w is the real wage and r is the real interest rate (depreciation is zero).

The interest rate is fixed at its steady state level. In the questions to follow assume in equilibrium each firm will employ one unit of labor, so l=1.

(a) Determine the firm's steady state capital stock as a function of τ.

(b) What is the steady state real wage?

(c) Determine $\partial k / \partial \tau$.

(d) Determine $\partial w / \partial \tau$.

Total taxes collected by the government are

$$T \equiv (\tau)[k_t^{\alpha} - w_t]. \qquad [2]$$

(e) Use this equation to determine $\partial \tau / \partial T$. (Remember k is a function of τ.)

(f) Cut total business taxes by 1 unit, ∂T =1. It has been argued that such a tax cut for business will not benefit workers. Evaluate this statement using the model. (Hint: Use the results from above to determine $\partial w / \partial T$ and then find δw when ∂T =1.)

(g) Assuming α =1/3 and τ =1/3, what is ∂w in (f)?

PROPOSAL MICRO QE, NOVEMBER 27, 2017

Problem 1 (50 minutes)
Consider two commodities with quantities x_1 and x_2. Further consider the two price-income combinations

$$(p_1, p_2, m) = (2, 1, 3),$$
$$(\widehat{p}_1, \widehat{p}_2, \widehat{m}) = (1, 1, 2).$$

(a) Determine the consumption bundle $(\overline{x}_1, \overline{x}_2)$ that satisfies
$p_1\overline{x}_1 + p_2\overline{x}_2 = m$ and
$\widehat{p}_1\overline{x}_1 + \widehat{p}_2\overline{x}_2 = \widehat{m}$.

(b) Draw in a diagram the "non-linear" budget set

$$B = \{(x_1, x_2) \in \mathbb{R}^2_+ \mid p_1x_1 + p_2x_2 \leq m,\ \widehat{p}_1x_1 + \widehat{p}_2x_2 \leq \widehat{m}\}.$$

(c) For arbitrary $c > 0$, $d > 0$, solve the utility maximization problem

$$\max\ x_1^c x_2^d \text{ s.t. } (x_1, x_2) \in B.$$

HINT. The solution can be obtained by solving the two problems

$\max\ x_1^c x_2^d$ s.t. $p_1x_1 + p_2x_2 \leq m$, $x_1 \geq 0$, $x_2 \geq 0$ and

$\max\ x_1^c x_2^d$ s.t. $\widehat{p}_1x_1 + \widehat{p}_2x_2 \leq \widehat{m}$, $x_1 \geq 0$, $x_2 \geq 0$

in a first step.

Problem 2 (30 minutes)
In the following simultaneous-move games Γ and $\widehat{\Gamma}$, only pure strategies are considered.

Γ

		PLAYER 2 L	C	R
PLAYER 1	T	4, 1	0, 0	6, 0
	M	2, 1	1, 3	3, 1
	B	1, 4	0, 1	5, 5

$\widehat{\Gamma}$

		PLAYER 2 L	C	R
PLAYER 1	T	4, 1	0, 0	6, 0
	M	2, 1	1, 3	3, 1
	B	1, 2	0, 1	5, 5

(a) Suppose that the simultaneous-move game Γ is played twice, with the outcome of the first stage observed by both players before the second stage begins. There is no discounting. Is there a (pure-strategy) subgame perfect Nash equilibrium [SPNE] such that the payoff (5,5) is achieved in the first stage? If your answer is YES, provide the details of such an SPNE and show that it is an SPNE, indeed. If your answer is NO, explain your answer carefully.

(b) Suppose that the simultaneous-move game $\widehat{\Gamma}$ is played twice, with the outcome of the first stage observed by both players before the second stage begins. There is no discounting. Is there a (pure-strategy) subgame perfect Nash equilibrium [SPNE] such that the payoff (5,5) is achieved in the first stage? If your answer is YES, provide the details of such an SPNE and show that it is an SPNE, indeed. If your answer is NO, explain your answer carefully.

Problem 3 (30 minutes)
Examine the monopolistic competition between two local pizzerias supplying the same neighborhood. The two owners choose their prices simultaneously; and the respective sales associated with any price profile $(p_1, p_2) \geq (0, 0)$ are

$$\begin{cases} q_1(p_1, p_2) = \max(30 - 2p_1 + p_2, 0); \\ q_2(p_1, p_2) = \max(30 - 2p_2 + p_1, 0). \end{cases}$$

In addition, assume that the unit cost of production is constant and given by c for both businesses (with $0 \leq c \leq 30$).

(a) Determine the best response function of each pizzeria.

(b) Are the prices p_1, p_2 strategic substitutes or strategic complements?

(c) Derive the Nash equilibrium of this model.

Problem 4 (70 minutes)
There is a private ownership economy $\mathcal{E} = \{(X^i, \succsim^i)_{i=1}^I, (Y^1), (\omega^i, \theta^i)_{i=1}^I\}$ with two consumers $i \in \{1, 2\}$, one firm, and two commodities $l \in \{1, 2\}$. Each consumer i chooses among commodity bundles in the set $X^i = \mathbb{R}^2_+$ according to his/her preferences $\succsim^i$ described by the Leontief utility function:

$$u(x_1^i, x_2^i) = \min\{2x_1^i, x_2^i\}.$$

The initial endowment of consumer 1 is given by $\omega^1 = (4, 2)$ and that of 2 by $\omega^2 = (4, 6)$. Both consumers have the same shares of firm 1; that is, $\theta_1^1 = \theta_1^1 = \frac{1}{2}$. Suppose that the firm can produce commodity 2 by using commodity 1 according to the following technology:

$$Y^1 = \{(-y_1, y_2) \in \mathbb{R}^2 : y_2 \leq 2 \cdot y_1, 0 \leq y_1\},$$

Let $p \in \mathbb{R}^2_+$, $p \neq 0$, be a price vector.

(*a*) Sketch the technology in a diagram and determine the firm's supply correspondence $y^1(p)$ as well as the profit function $\pi^1(p)$.

(*b*) Determine consumer i's Walrasian demand correspondence $x^i(p, \omega^i, \theta^i)$.

(*c*) Is the demand correspondence homogeneous of degree zero? If so, then use $p = (p_1, 1)$.

(*d*) Determine the Walrasian equilibrium for the economy $\mathcal{E}$.

(*e*) Characterize the set of all Pareto-optimal allocations in the economy $\mathcal{E}$.

(*f*) Suppose now that the only feasible production plans are those of free-disposal, i.e., $Y^1 = -\mathbb{R}^2_+$. What is the Walrasian equilibrium in this situation?