Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, August 15, 2016

Part 1: Microeconomics

3 Questions, 3 pages

Note: Each question is weighted equally.

Problem 1 (60 minutes)
Felix Chance has current wealth $W > 0$. Felix invests the amount C with $0 \leq C \leq W$ in a risky asset and the amount $W - C$ in a safe asset.

(A) Investing the amount C in the risky asset yields future income

1) $z_1 C$ with probability $p_1 > 0$,
2) $z_2 C$ with probability $p_2 > 0$,

...

n) $z_n C$ with probability $p_n > 0$

where $n \geq 2$, $0 \leq z_1 < z_2 < \ldots < z_n$ and $\sum_{i=1}^{n} p_i = 1$. Let $z = \sum_{i=1}^{n} p_i z_i$ be the expected per unit return of the risky investment.

(B) Investing the amount $W - C$ in the safe asset yields future income $s(W - C)$ with $s > 0$.

Felix's future wealth consists of his investment income. He is an expected utility maximizer with respect to future wealth, with von Neumann-Morgenstern utility function $u(w)$ for future wealth $w \geq 0$. u is twice differentiable with $u' > 0$ and $u'' < 0$.

(a) Write down the expression for $EU(C)$, Felix's expected utility of future wealth as a function of C.

(b) Derive $EU'(C)$, the derivative of EU with respect to C.

(c) Derive $EU''(C)$.

(d) What is the sign of $EU''(C)$ if $z \neq s$?

(e) Determine the sign of $EU'(0)$ by comparing z and s.

(f) When is it optimal for Felix to choose a positive C, that is to take some risk although Felix is risk averse?
HINT. Given the answer in (e), when is $C = 0$ not optimal?

(g) Suppose $W = 1, n = 2$, $z_1 = 0$, $z_2 = 1.8$, $p_1 = p_2 = 1/2$, $s = 0.8$, and $u(w) = \sqrt{w}$ for $w \geq 0$.
Determine the value of C that maximizes $EU(C)$.

Problem 2 (60 minutes)
Consider a Cournot duopoly with firms 1 and 2 which produce a homogeneous product. Firm i has cost function $C_i(q_i) = a_i \cdot q_i^2$ for its output $q_i \geq 0$ where $a_1 > a_2 \geq 0$. They face the market inverse demand function

$$P(q) = \begin{cases} 1 - q & \text{if } 0 \leq q \leq 1; \\ 0 & \text{if } q > 0. \end{cases}$$

(a) **Determine** the Cournot equilibrium quantities and profits.

(b) Suppose firm 2 drops out of the market.
Determine firm 1's optimal output and profit as a monopolist.

(c) Suppose firm 1 acquires firm 2 so that it owns and can use both production facilities.

(c.1) For $q > 0$, **determine** $C(q)$, firm 1's minimal cost of producing output q after it has acquired firm 2.

(c.2) **Determine** the profit maximizing output and maximum profit of the merged firm.

Problem 3 (60 minutes)

(A) For the following preferences, determine the Walrasian demand function $x(p, w)$, where p denotes the vector of positive prices and $w \geq 0$ is the consumer's wealth.

$$u(x_1, x_2, x_3) = \min(2x_1, x_2, x_3), \quad \text{for any } (x_1, x_2, x_3) \in \mathbb{R}^3_+.$$

Is this demand function homogeneous of degree zero in (p, w)? Does it satisfy Walras' law?

(B) Consider the two-commodity setting where the consumer's utility function is given by

$$v(x_1, x_2) = \left(x_1^{1/2} + 3x_2^{1/2}\right)^2, \quad \text{for any } (x_1, x_2) \in \mathbb{R}^2_+.$$

(i) Write the utility maximization problem and compute the Walrasian demand function $x(p_1, p_2, w)$. Explain carefully.

(ii) Show that the indirect utility function can be written in the form $v(p_1, p_2, w) = w\left(\frac{\alpha_1}{p_1} + \frac{\alpha_2}{p_2}\right)$, where α_1 and α_2 are positive numbers.

(iii) Use the indirect utility function and Shephard's lemma to derive $h(p, \mu)$, the Hicksian demand function.

Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, August 18, 2016

Part 2: Econometrics and Macroeconomics

Part 2A: Econometrics (3 Questions on 5 pages)
Part 2B: Macroeconomics (1 Question on 1 pages)

Note: Econometrics counts for 60% of this exam (with each question being weighted equally) and macro for 40%.

Question 1 (20 minutes)

(a) In the context of the simple (one parameter) Normal model:

The simple (one parameter) **Normal model**		
Statistical GM:		$X_t = \mu + u_t,\ t\in\mathbb{N}$,
[1]	Normal:	$X_t \backsim \mathsf{N}(.,.)$,
[2]	Constant mean:	$E(X_t) = \mu$, for all $t\in\mathbb{N}$,
[3]	Constant variance:	$Var(X_t) = \sigma^2$, for all $t\in\mathbb{N}$,
[4]	Independence:	$\{X_t,\ t\in\mathbb{N}\}$ - independent process.

where σ^2 is known, explain how the following $(1-\alpha)$ Confidence Interval (CI):

$$\mathbb{P}\left(\overline{X}_n - c_{\frac{\alpha}{2}}(\tfrac{\sigma}{\sqrt{n}}) \leq \mu \leq \overline{X}_n + c_{\frac{\alpha}{2}}(\tfrac{\sigma}{\sqrt{n}}); \mu{=}\mu^*\right) = 1-\alpha, \tag{1}$$

is constructed.

(b) How should one interpret this CI using the long-run metaphor, where one imagines collecting N sample realizations of size n and evaluates the CI for each realization. Explain the difference between the probabilistic statement in (1) and the long-run metaphor.

(c) Explain what is wrong with the statement that the observed CI has probability $1-\alpha$, i.e.

$$\mathbb{P}\left(\overline{x}_n - c_{\frac{\alpha}{2}}(\tfrac{1}{\sqrt{n}}) \leq \mu \leq \overline{x}_n + c_{\frac{\alpha}{2}}(\tfrac{1}{\sqrt{n}}); \mu{=}\mu^*\right) = 1-\alpha,$$

where $\overline{x}_n$ denotes the observed value of $\overline{X}_n$.

(d) Explain how the above CI relates to the test of the hypotheses:

$$H_0\colon\ \mu{=}\mu_0 \text{ vs. } H_0\colon\ \mu{\neq}\mu_0,$$

where μ_0 is a given value, paying particular attention to how the underlying reasoning underlying CI and hypothesis testing differ.

Question 2

Suggested time: 20 minutes

Consider a regression of "stress level" for young mothers on a set of explanatory variables:

$$y_i = \mathbf{x}_i'\boldsymbol{\beta} + s_i\gamma + \epsilon_i, \quad \epsilon_i \sim n\left(0, \sigma_\epsilon^2\right) \tag{1}$$

where y_i is some continuous, unbounded clinical measure of stress, $\mathbf{x}_i$ includes a set of exogenous regressors, and s_i measures the average daily amount of sleep for individual i, expressed in hours (allowing for fractional hours). The error term has the usual CLRM properties.

Unbeknownst to the researcher, the sleep variable s_i follows a second regression model:

$$s_i = \delta y_i + \eta_i \tag{2}$$

where η_i is a "well-behaved" CLRM error term. Assume that $\mathbf{x}_i$, ϵ_i, and η_i are uncorrelated with one another.

Part (a)
Express s_i as a function of all other components in (1) and (2), except for y_i.

Part (b)
Derive the true covariance between s_i and ϵ_i.

Part (c)
Let the full regression model in (1) for the entire sample of n observations be written as

$$\begin{aligned} \mathbf{y} &= \mathbf{M}\boldsymbol{\theta} + \boldsymbol{\epsilon}, \quad \text{where} \\ \mathbf{M} &= \begin{bmatrix} \mathbf{X} & \mathbf{s} \end{bmatrix} \\ \boldsymbol{\theta} &= \begin{bmatrix} \boldsymbol{\beta} \\ \gamma \end{bmatrix} \end{aligned} \tag{3}$$

Derive $cov\left(\mathbf{s}, \boldsymbol{\epsilon}\right) = E\left(\mathbf{s}'\boldsymbol{\epsilon}\right)$, using your result from part (b).
(Hint: $cov\left(\epsilon_i, s_i\right) = E\left(\epsilon_i s_i\right)$.)

Part (d)
Using the partitioned form of the OLS estimator for $\boldsymbol{\theta}$ (call it $\hat{\boldsymbol{\theta}}$) in terms of $\mathbf{X}$ and $\mathbf{s}$, show that it is biased. (Hint: You do NOT need to solve the partitioned inverse matrix involved in this operation.)

Part (e)
Describe, in words, how one could use an instrumental variable approach and Two-Stage-Least-Squares (TSLS) estimation to overcome this endogeneity problem. Can you think of one or two good instruments?

Question 3

(suggested time: 20 min.)

Read the appended article, with scatterplot, from a recent issue of *The Economist* magazine.

Name three potential problems – two related to *statistical misspecification* and one related to *substantive misspecification* – with the implicit well-behaved classical bivariate linear regression model (of PISA score vs. national per capita ice cream consumption) for which *The Economist* is quoting an R^2 value.

Briefly discuss the impact of each of these problems on consistent OLS estimation of the coefficient on ice cream consumption and on valid statistical inference – confidence intervals and/or hypothesis testing – with regard to this coefficient.

[Note: The graphic in the article is given in larger form on the last page of this exam]

The Economist

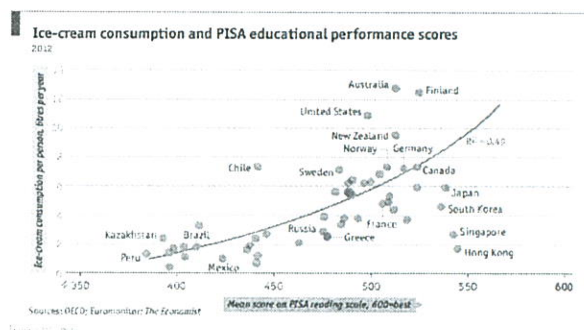
Graphic detail

Charts, maps and infographics

Daily chart

Ice cream and IQ

Apr 1st 2016, 10:37 by THE DATA TEAM

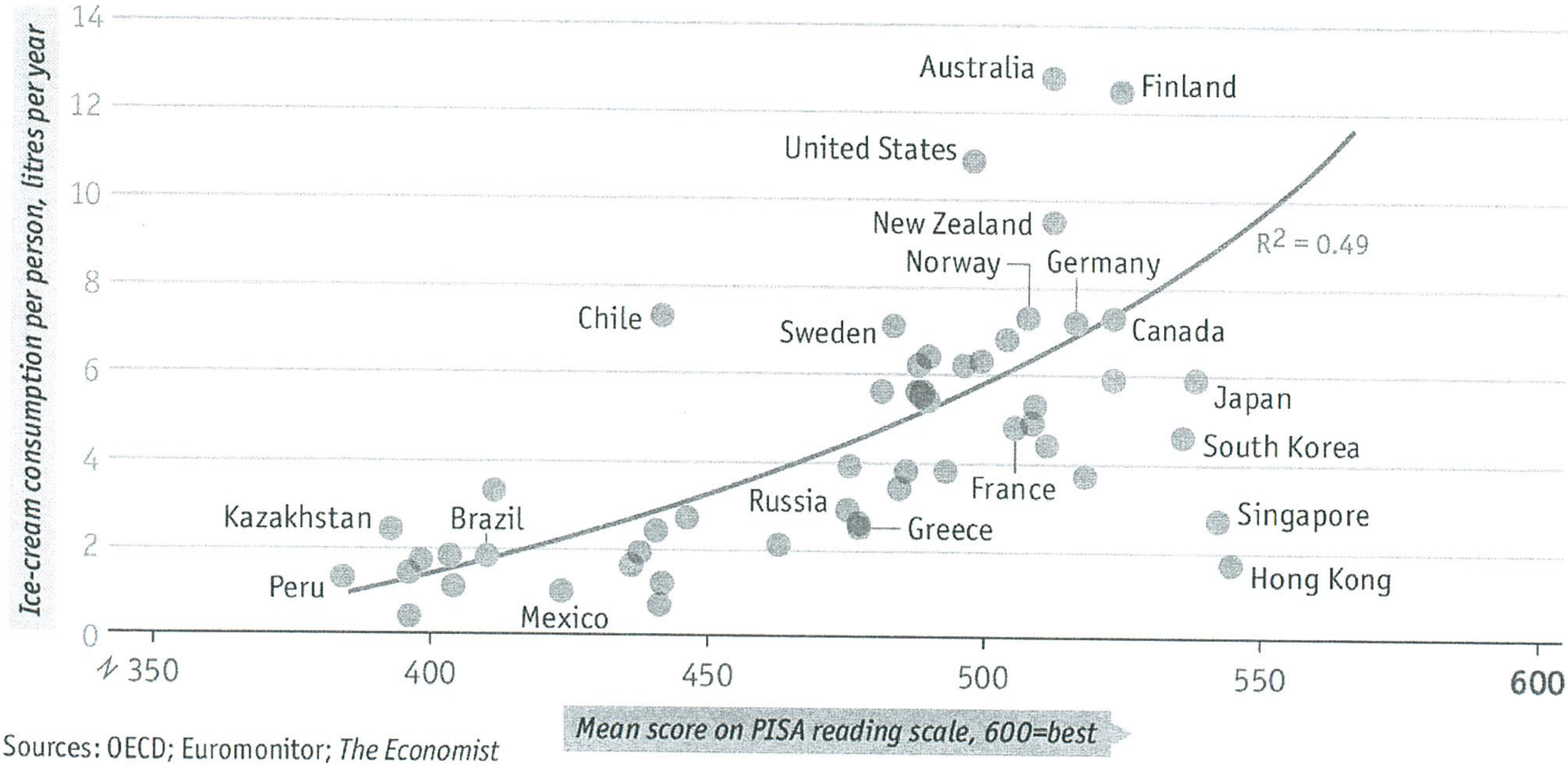


WITH school-exam season just around the corner, parents will be increasingly preoccupied with how to make their children sit down, keep quiet and study. Some will purchase hefty revision guides while others will turn to tutors for help. However number-crunching from *The Economist* might offer a rather sweeter solution. Ice cream consumption, it seems, has a strong relationship with reading ability, based on the OECD's PISA educational performance scores. Australia, for instance, scoffs 13 litres of gelato per year—more than any other country—and its children are among the most literate in the world. And it is not just sun-kissed states that show such a striking correlation. Finland, Canada and Sweden all top the PISA rankings and are avid consumers of frozen desserts. At the other end of the counter, an average Peruvian puts away barely a litre of the cold stuff each year and comes last in the rankings. Ice cream, it would appear, induces the opposite of "brain-freeze" in students.

There are, of course, outliers. Suitably Chile (by name and nature) eats a large amount of ice cream, yet that has had a mysteriously small effect on literacy. In well-off Asian countries, by contrast, children are book-rich but ice-cream poor. Such findings have tasty policy implications for parents and politicians alike. Though it may seem like an odd suggestion on a brisk early-April morning, year-round subsidised ice-cream for children could improve educational attainment. And ice-cream vans should park closer to libraries to help boost reading skills too. What a scoop.

Ice-cream consumption and PISA educational performance scores

2012



Sources: OECD; Euromonitor; *The Economist*

Macro Question

There is a continuum of farmers and gatherers who live forever. Both types are risk neutral and only care about discounted amount of fruit consumption, but farmers have a discount factor of 0.7 while gatherers have a discount factor of 0.8. The population size of farmers is 1 and the population size of gatherers is also 1.

There is fixed amount of land of 1 acre, and land is used to produce fruit (which cannot be stored). Fruit is the numeraire.

Each farmer can use his land k_t to produce fruit in the next period with the technology $y_{t+1} = 2k_t$, but only half of the fruit produced is tradable (the farmer just eats the rest of it).

Each gatherer can use her land k_t to produce fruit in the next period with the technology $G(k_t) = (k_t + 0.01)^{1/2}$, and all the fruit produced is tradable.

There is a credit market for trading 1 unit of fruit today for R unit of fruit tomorrow. There is also a spot market of land, and each acre of land has a price of q_t.

Each farmer faces a <u>collateral constraint</u> that their debt repayment tomorrow cannot exceed the value of his land tomorrow.

We consider the perfect foresight equilibrium with no bubble, and you are told that in equilibrium farmers borrow and invest in land as much as possible.

(1) Write down the budget constraints for the two types of people.

(2) Solve for the equilibrium and explain intuitively why R=1.25 in equilibrium.

(3) Calculate the steady state values of land price, and landholding and borrowing by farmers.

(4) How much fruit in total is produced in the steady state?

(5) Now there is no collateral constraint. How much fruit in total is produced in the steady state?

(6) Explain intuitively why the answers to (4) and (5) are different.