Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, January 11, 2016

Part 1: Microeconomics

Note: The minutes assigned to each question indicate the weight given each question by the graders.

MICRO QE, JANUARY 2016

Problem 1 (40 minutes)

Suppose that a union is the sole supplier of labor to all firms in a given oligopoly (such as the United Auto Workers is to General Motors, Ford, and others). The inverse demand (giving the market-clearing price as a function of aggregate output) is

$$P(Q) = \begin{cases} \alpha - Q, & \text{if } Q \leq \alpha; \\ 0 & \text{otherwise.} \end{cases}$$

To keep things simple, assume that the firms have no costs other than wages; and the production function for every firm $i \in \{1, \ldots, n\}$ is $q_i = L_i$ (that is, output equals labor). Hence, we also have $Q = L$, where $L = L_1 + \ldots + L_n$ stands for total employment in the unionized firms.

The dynamic game between the union and the firms is described as follows.

1. The union makes a wage demand, w, which applies to all firms.

2. The firms observe (and accept) w and then simultaneously choose their employment levels, L_i for each firm $i \in \{1, \ldots, n\}$.

3. The payoffs are determined: the union's utility is $(w - w_a)L$, where w_a is the wage that union members can earn in alternative employment; each firm i's profit is $(P(L) - w)\, L_i$.

(a) Use backward induction to derive the subgame perfect outcome of this game (assume that $0 \leq w_a < \alpha$).

(b) Compute the union's utility under the subgame perfect outcome. How does it change with the number of firms? Interpret this result.

(c) Compute the equilibrium profit of each firm; and then discuss its relationship to the union members' alternative wage (w_a).

Problem 2 (50 minutes)

Suppose that there are two types of consumers (H and L) for a firm's product. The proportion of consumers of type L is $\lambda \in (0, 1)$. A consumer of type $i \in \{H, L\}$ enjoys the utility $u_i(x, T) = \theta_i v(x) - T$ when consuming the quantity $x \in [0, 1]$ of the good and paying a total amount of T for it, where

$$v(x) = \frac{1 - (1 - x)^2}{2}.$$

The firm is the sole producer of this good; and its cost per unit is c, with $0 < c < \theta_L < \theta_H$.

(a) Assuming a linear tariff ($T = px$), determine the optimal price p charged by a non-discriminating monopolist. Under which conditions will the monopolist choose to exclude the consumers of type L?

(b) Consider a monopolist that can distinguish the two types (using some observable characteristic) but may only charge a linear tariff to each type ($T_i = p_i x_i$, for $i = H, L$). Characterize the optimal prices p_H and p_L.

(c) Compute the fully optimal nonlinear tariffs and the corresponding quantities. Interpret your results.

Problem 3 (45 minutes)

PART A

Consider the case of two goods and a consumer with utility representation

$$U(x_1, x_2) = \begin{cases} 2x_1 + x_2 \text{ if } x_2 \geq 2x_1 \geq 0; \\ \frac{4}{3}x_1 + \frac{4}{3}x_2 \text{ if } 2x_1 \geq x_2 \geq \frac{1}{2}x_1 \geq 0; \\ x_1 + 2x_2 \text{ if } \frac{1}{2}x_1 \geq x_2 \geq 0. \end{cases}$$

(a) Professor Klugmann claims that the consumer has homothetic preferences. Is the professor correct? Explain!

(b) Let $p_1 > 0, p_2 > 0, m > 0$.

Solve the consumer's utility maximization problem

$$\max\ U(x_1, x_2) \text{ subject to } p_1x_1 + p_2x_2 \leq m.$$

(A diagram with some indifference curve(s) might help.)

(c) Find $p_1 > 0, p_2 > 0, m > 0$ so that $(x_1, x_2) = (150, 50)$ is a solution of the consumer's utility maximization problem.

PART B

Consider a pure exchange economy with 15 consumers. Each consumer $i = 1, \ldots, 15$ has an endowment bundle $\omega_i \in \mathbb{R}^2_+$ and the utility representation given in PART A. Suppose $\omega_1 = (5, 5)$ and $\omega = (150, 50)$ where $\omega = \sum_{i=1}^{15} \omega_i$ is the aggregate endowment.

(d) Explain why the answer to (c) provides a Walrasian equilibrium price system (p_1^*, p_2^*).

(e) Determine consumer 1's equilibrium consumption!

Problem 4 (45 minutes)

Consider a pure exchange economy with two commodities $L = \{1, 2\}$ and two consumers indexed by $i \in \{A, B\}$. Consumers' preferences over $\mathbb{R}^L_+$ are represented be the following utility functions:

$$u^A(x_1^A, x_2^A) = \min\{2x_1^A + x_2^A, 3x_2^A\},$$
$$u^B(x_1^B, x_2^B) = \min\{x_1^B, x_2^B\}.$$

The consumers' initial endowments are given by $(\omega_1^A, \omega_2^A) = (3, 3)$ and $(\omega_1^B, \omega_2^B) = (6, 3)$. Let p_1 denote the price of good 1 and let p_2 denote the price of good 2.

1. Consider consumer A. Draw some indifference curves in a diagram. Determine the marginal rate of substitution. Derive A's demand correspondences $x_1^A(p_1, p_2, \omega^A)$ and $x_2^A(p_1, p_2, \omega^A)$.

2. Derive B's demand correspondences $x_1^B(p_1, p_2, \omega^B)$ and $x_2^B(p_1, p_2, \omega^B)$.

3. Draw the Edgeworth box for this economy. Derive the set of all Pareto optimal allocations and illustrate the set in the Edgworth box.

4. Compute the competitive market equilibrium for this exchange economy.

5. Which of the Pareto-optimal allocations can be supported by a price equilibrium with transfers? Determine the prices p_1 and p_2, the distribution of wealths m^A and m^B as well as the transfers T^A and T^B.

Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, January 14, 2016

Part 2: Econometrics and Macroeconomics

Part 2A: Econometrics (3 Questions, 90 minutes)
Part 2B: Macroeconomics (3 Questions, 90 minutes)

Note: The minutes assigned to each question indicate the weight given each question by the graders.

Econometrics, Question 1

(30 min.)

Let

$$Y = X\beta + \varepsilon \qquad \text{with} \qquad \varepsilon \sim NIID(0, \sigma^2)$$

where it is assumed that $E\{X\} = 0$ if X is not fixed in repeated samples.

1. Derive an expression for the OLS fitting errors, e, in terms of ε.

2. Show that $E\{e\} \neq 0$ unless either X is fixed in repeated samples or X is distributed independently from ε.

3. Demonstrate that $\text{cov}(X, e) = E\{X^t e)$. {Recall that it is assumed that $E\{X\} = 0$ if X is not fixed in repeated samples.}

4. Show that $X^t e = 0$ regardless of whether X is fixed in repeated samples, independent of ε, or, in fact, regardless of anything beyond the definitional assumption made here that e is the vector of OLS fitting errors for the model. Therefore, since $X^t e$ is a non-random variable equal to fixed vector of zeroes, its expectation, $E\{X^t e\}$, must be zero. Hence, from the result in part 3, $\text{cov}(X, e)$ is inherently zero.

5. What does this result (in part 4) imply about the potential usefulness of the OLS fitting errors in testing the proposition that X is uncorrelated with ε, even with an arbitrarily large sample?

6. Demonstrate why the proposition that X is uncorrelated with ε matters for consistent estimation of β. In particular, demonstrate – presuming positive definiteness for the asymptotic variance of X – that the OLS estimator of β is consistent only if X is asymptotically uncorrelated with ε. Hint: you may identify the asymptotic variance of X as $\text{plim}\{(1/N)\, X^t X\}$ and the asymptotic $\text{cov}(X, \varepsilon)$ as $\text{plim}\{(1/N)\, X^t \varepsilon\}$.

Econometrics, Question 2

(30 min.)

(a) "When modeling cross-section data one need not worry about heterogeneity or dependence." Discuss this claim and contrast it to the alternative view that "all data, cross-section, time-series or panel, can be viewed as realizations of generic stochastic processes where heterogeneity and dependence are relevant because there always orderings of interest (or they should be) for any data"; give examples if it helps.

(b) For the Fixed Effects Panel (FEP) data model:

Table 3: Fixed Effects Panel (FEP) model
$y_{it}=\boldsymbol{\beta}^{\top}\mathbf{x}_{it}+c_i+\epsilon_{it}, i\in\mathbb{N},\ t\in\mathbb{T},$
[i] $E(\epsilon_{it})=0$, [ii] $E(\epsilon_{it}^2)=\sigma_{\epsilon}^2$,
[iii] $E(\epsilon_{it}\epsilon_{js})=0$, for $t\neq s,\ i\neq j,\ i,j\in\mathbb{N},\ t,s\in\mathbb{T}$

derive the OLS estimator $\widehat{\boldsymbol{\beta}}_{OLS}$ of $\boldsymbol{\beta}$ $(m\times 1)$ as a within-groups estimator using the T-averages form:

$$\rightarrow\ \overline{y}_i=\boldsymbol{\beta}^{\top}\overline{\mathbf{x}}_i+c_i+\overline{\epsilon}_i,$$

where $\overline{\mathbf{x}}_i=\frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_{it}$, $\overline{y}_i=\frac{1}{T}\sum_{t=1}^{T}y_{it}$, $\overline{\epsilon}_i=\frac{1}{T}\sum_{t=1}^{T}\epsilon_{it}$, $i=1,2,...,N$ are the within-groups sample means.

(c) Compare and contrast the conditions under which the estimators $\widehat{\boldsymbol{\beta}}_{OLS}$ and $\widehat{c}_i=\overline{y}_i-\widehat{\boldsymbol{\beta}}_{OLS}^{\top}\overline{\mathbf{x}}_i$, $i=1,2.\ ...,N$, are consistent for $\boldsymbol{\beta}$ and c_i, respectively. Explain your answer.

(d) Explain why the "neglected individual heterogeneity" is appropriate for the fixed effects term c_i, when the latter refers to a mean $i-$heterogeneity of the process $\{\mathbf{Z}_{it}:=(y_{it},\mathbf{X}_{it}),\ i\in\mathbb{N},\ t\in\mathbb{T}\}$, assumed to be Normal, Independent, covariance stationary, but mean $i-$heterogeneous. Relate your answer to the estimator $\widehat{c}_i$ in (c).

Econometrics Question 3

30 minutes

Consider the Poisson model for a random variate y with parameter λ, given as

$$\begin{aligned} p\left(y|\lambda\right) &= \frac{\lambda^{y} exp(-\lambda)}{y!}, \quad \text{with} \\ E\left(y|\lambda\right) &= V\left(y|\lambda\right) = \lambda, \quad \lambda > 0, y \in \{0, 1, 2, 3, ...\} \end{aligned} \tag{1}$$

Part (a)

Now consider a sample of n observations from this distribution, with each observation generically labeled $y_i, i = 1 \ldots n$. Write down the joint distribution for the sample data (in *un*-logged form). Call it $p\left(\mathbf{y}|\lambda\right)$.

Part (b)

Suppose you stipulate a *gamma* prior density for λ with shape parameter a and inverse scale ("rate") parameter b, given as

$$\begin{aligned} p\left(\lambda\right) &= g\left(a, b\right) = \frac{b^{a}}{\Gamma\left(a\right)} \lambda^{(a-1)} exp\left(-b\lambda\right), \quad \text{with} \\ E\left(\lambda\right) &= \frac{a}{b}, \; V\left(\lambda\right) = \frac{a}{b^{2}}, \quad \lambda, a, b > 0, \end{aligned} \tag{2}$$

Show that the posterior distribution of λ, given your collected data from the Poisson, is also a gamma. Show the form of the posterior shape and rate parameters (you can call them a^* and b^*).

Part (c)

Show that the posterior expectation can be written as a weighted average of the prior expectation and the sample mean. What happens to this posterior expectation as $n \to \infty$?

Part (d)

Suppose you are an investor on "*Shark Tank*" (popular TV show where wealthy entrepreneurs are solicited to finance new products), and you are asked to invest in a product with weekly sales of y units. The inventor (person who is asking you for money) reports that, at the beginning of last year, he expected to sell 10 units per week with a variance of 5 units. He then sold 898 units over the entire year, that is over 52 weeks. He shows you a histogram chart of weekly sales, and it looks exactly like a Poisson distribution.

You quickly realize that the inventor's prior expectation and variance correspond to a gamma with shape $a = 20$ and rate $b = 2$. Letting the actual weekly sales follow a Poisson with parameter λ, you quickly compute the posterior expectation and variance for weekly sales. You would be comfortable investing in this deal if you could expect at least 20 sales per week for the coming year. Would you invest or not? Explain.

Now suppose the inventor had sold all 818 units in 40 weeks instead of 52. How, if at all, would this change your decision? Explain.

Three Macroeconomics Questions 30 minutes each.

[1] Consider the Ramsey model with zero population growth and no technological progress. Output is Ak_t^{α}. At time t the government has purchases equal to g_t. Spending is financed with an income tax and a lump sum tax. The tax rate is constant at τ and the lump sum tax is z_t, so $\tau Ak_t^{\alpha}+z_t=g_t$. Government purchases are a constant fraction of output given by $\tau Ak_t^{\alpha}+z_t=g_t=\bar{g}Ak_t^{\alpha}$. The individual's budget constraint is

[i] $\dot{k}_t=[A(1-\tau_t)k_t^{\alpha}-\delta k_t-z_t-c_t]$,

The individual seeks to solve

[ii] $\max\int_0^{\infty}[c_t^{1-\gamma}/(1-\gamma)]\exp(-t\theta)dt$, s.t. [*i*], given k_0.

(a) What is the optimal value for $\frac{\dot{c}_t}{c_t}$?

(b) What are the steady state values for *c* and *k*? Call them *c** and *k**.

(c) Draw the phase diagram for this problem. Assume k_0<*k** and indicate the time path to the steady state.

(d) Suppose there is a increase in τ offset by an decrease z_t, leaving total taxes at each time t unchanged. Show the effect on the phase diagram, indicating the path to the new steady state. What happens to *c** and *k**? How would this change impact individual utility in the long-run?

[2] Freedonia is an endowment economy with each individual receiving output of y_t at time t. Individuals can borrow and lend at a fixed world interest rate of θ. Bond holdings for an individual are b_t. (If b_t>0, the individual has been a net lender in the past; if b_t<0, a net borrower in the past.) The budget constraint is

$$\dot{b}_t = \theta b_t + y_t - c_t .$$

Given this budget constraint, the individual seeks to maximize utility as given by

$$\int_0^\infty [(\ln c_t)\exp(-t\theta)dt .$$

(a) What is the optimal time path of consumption?

Assume b_0=0 and income is given by

$$y_t = y_0 \text{, for } 0 \le t < \bar{t} \text{ ;}$$
$$y_1 < y_0 \text{, for } t \ge \bar{t} \text{ .}$$

(b) Determine the time 0 present value of income for an individual. What is the optimal value for c_0?

(c) Determine the time 0 value of permanent income, as defined by Friedman. Call it y^P. What is time 0 consumption as a function of y^P?

(d) Given your time path of consumption what is the value of $b_{\bar{t}}$ ($b(\bar{t})$)? What is the time path of b_t after time $\bar{t}$? Explain.

[3] To answer the following questions assume real output and the real interest rate are constant at $\bar{y}$ and $\bar{r}$. Consider an equilibrium in which long-run inflation is also a constant and in which inflation and expected inflation are equal. In this setting, *the fiscal theory of the price level* will determine the time zero price level and the nominal interest rate, $R\ (=\bar{r}+\pi)$, with the following equations:

$$M_0 / P_0 = \Phi(R);$$
$$[B_0^g + M_0]/P_0 = S^f + S^s(R).$$

M_0 is the time 0 nominal money supply, P_0 is the time 0 price level, B_0^g is the time 0 nominal supply of government bonds, S^f is the present value of future government fiscal surpluses, S^s is seigniorage (an increasing function of R), and Φ is the real demand for money (a decreasing function of R).

Use this model to determine whether each of the following statements is true or false. If the answer to any question is ambiguous, depending upon unspecified circumstances, explain why this is so.

(a) A temporary nominal tax cut at time 0 financed with additional money is inflationary.

(b) An increase in B_0^g (caused, say, by a time 0 tax cut) is inflationary.

(c) An open market purchase of government bonds by the monetary authority is equivalent to a helicopter drop of money.