Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, March 14, 2016

Part 1: Microeconomics

3 Questions, 5 pages

Note: The minutes assigned to each question indicate the weight given each question by the graders.

MAKE-UP QE, MARCH 2016

Problem 1 (70 minutes)

Consider a world with $L > 1$ commodities so that the commodity space is $\mathbb{R}^L$. Commodities are denoted $k \in \{1, \ldots, L\}$ and commodity bundles are denoted $x = (x^1, x^2, \ldots, x^L) \in \mathbb{R}^L$.

You can use without proof:

> If a Cobb-Douglas consumer j has income $m_j > 0$ and utility representation
>
> $$u_j(x_j) = \beta_j^1 \ln x_j^1 + \beta_j^2 \ln x_j^2 + \cdots + \beta_j^L \ln x_j^L$$
>
> with $\sum_k \beta_j^k = 1$, then j's demand for good k at the price system $p = (p^1, \ldots, p^L \gg 0$ is $x_j^k(p; m_j) = \beta_j^k \cdot \dfrac{m_i}{p^k}$.

Now let the economy consist of $n > 1$ consumers $i = 1, \ldots, n$, each with endowment $\omega_i = \omega_0$ and utility representation

$$u_i(x_i) = a_i^1 \ln x_i^1 + a_i^2 \ln x_i^2 + \ldots + a_i^L \ln x_i^L$$

where $\omega_0 = (\omega_0^1, \ldots, \omega_0^L) \gg 0$ and $a_i^1 > 0, \ldots, a_i^L > 0$, $\sum_k a_i^k = 1$.

(a) For each good k and price system $p \gg 0$, determine the aggregate demand $x^k(p) = \sum_{i=1}^n x_i^k(p; \omega_i)$ in this economy.

(b) Define $b_I^k = (\sum_{i=1}^n a_i^k)/n$ for each good k and price system $p \gg 0$. Consider a consumer I with endowment $\omega_I = n \cdot \omega_0$ and utility representation

$$u_I(x_I) = b_I^1 \ln x_I^1 + b_I^2 \ln x_I^2 + \ldots + b_I^L \ln x_I^L.$$

Determine $x_I^k(p; \omega_I)$, consumer I's demand for each good k given the system $p \gg 0$.

(c) What does the comparison of your answers in (a) and (b) show? HINT. Do you have any recollection of the "representative consumer"?

(d) Find a market clearing price system for the n-consumer economy! HINT. You can either determine market clearing prices from the market clearing conditions (assuming $p\omega_0 = 1$) or use the correct answer in (c).

(e) Let $L = 3$, $n = 3$, $\omega_0 = (2, 1, 1)$,
$(a_1^1, a_1^2, a_1^3) = (1/2, 1/4, 1/4)$,
$(a_2^1, a_2^2, a_2^3) = (1/4, 1/2, 1/4)$,
$(a_3^1, a_3^2, a_3^3) = (1/4, 1/4, 1/2)$.
Determine the Walrasian equilibrium allocation of the economy!

Problem 2 (50 minutes)

Two players are bargaining over a pie of initial size $s = 1$. Player 1 moves first by offering a portion $x \in [0, 1]$ to Player 2. If this first offer is accepted, the game ends and the payoffs are $1 - x$ for Player 1 and x for Player 2. To capture the effects of discounting, assume that the pie is subject to decay and, in case Player 2 rejects the first offer, the residual value of the pie is $\delta \in (0, 1)$. Following her refusal, Player 2 has to make a counteroffer by proposing an amount $y \in [0, \delta]$ to player 1. If player 1 accepts the counteroffer, she gets y and Player 2 gets $\delta - y$. Otherwise, the players both get 0. Each person cares only about the amount she receives, and would like to receive as much as possible.

(a) Give a schematic game-tree representation of this bargaining process.

(b) Use backward induction to derive a **subgame perfect Nash equilibrium** of this game (clearly specify the strategies and the outcome).

(c) Describe the **subgame perfect outcome** for the variant where the two players are allowed to repeat the above bargaining process up to $T \geq 2$ times (as long as an offer has not been accepted), with the pie losing a fraction $1 - \delta$ of its value following every refusal (by either player). If all $2T$ offers have been rejected then the game ends and the players each get 0. *What is the limit of this equilibrium outcome as T goes to infinity?*

Problem 3 (60 minutes)

Consider a consumer who consumes two goods, leisure with quantity x_1 and a single consumption good (that stands for all other goods) with quantity x_2. The consumer is endowed with a positive amount ω_1 of time and zero amount of the consumption good, that is, the endowment bundle is $\omega = (\omega_1, 0)$ with $\omega_1 > 0$. Suppose that the price system assumes the form $p = (p_1, p_2) = (p_1, 1)$ which means that the consumption good serves a numéraire and p_1 is both the nominal and the real wage rate. Then the consumer's budget line is as in the above diagram. Suppose further that the tangency condition holds at the consumer's optimal consumption bundle (x_1^*, x_2^*), the solution of the problem

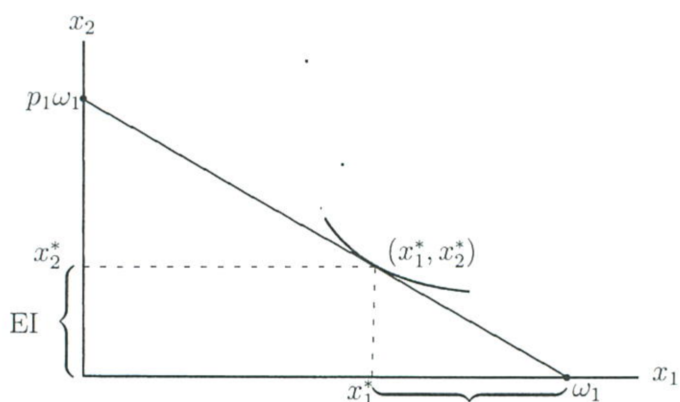
$$\max_{(x_1, x_2)} u(x_1, x_2) \text{ s.t. } px = p\omega,$$

where we assume that the budget constraint is binding. (x_1^*, x_2^*) is also shown in the diagram. Moreover,

$\ell = \omega_1 - x_1^*$ is the consumer's labor supply;

$EI = p_1 \ell$ is the consumer's labor income or "earned income".

Notice that $EI = x_2^*$ because of $p_2 = 1$. In general, EI is less than $p\omega = p_1\omega_1$, the income that enters the budget constraint.



The US Earned Income Tax Credit or Earned Income Credit (EIC) for low-income workers works like a negative income tax. Instead of paying taxes, they receive a subsidy from the government. For a couple with one child filing jointly or a single head of household with one child, the terms are roughly as follows:

EIC is 30% of earned income for income up to \$10,000 where it reaches its maximum of \$3,000. Then EIC remains constant until earned reaches \$20,000. From thereon, EIC is phased out at a 15% rate until earned income reaches \$40,000 at which point EIC=0. Thus, omitting the \$ symbol from now on:

$$EIC = \begin{cases} 0.3 \cdot EI & \text{for} \quad EI \leq 10,000; \\ 3,000 & \text{for} \quad 10,000 \leq EI \leq 20,000; \\ 0.15 \cdot (40,000 - EI) & \text{for} \quad 20,000 \leq EI \leq 40,000; \\ 0 & \text{if} \quad EI \geq 40,000. \end{cases}$$

Your Task is to determine the impact of this tax credit on a consumer's labor supply, whether the tax credit causes the worker to supply more, less or the same amount of labor than in the absence of the tax credit. To answer this question, consider a worker with $\omega = (\omega_1; 0) = (50,000; 0)$ and the price system $p = (p_1, p_2) = (1, 1)$. Then $EI = \ell$ in the above diagram. However, ℓ and EI may change when the EIC is applicable and you have to find out in which direction they change.

(a) Draw a large diagram with that particular consumer's budget line without EIC.

(b) Draw in the same diagram the consumer's budget constraint in the presence of EIC.

In the following cases, first determine the consumer's optimal consumption (x_1^*, x_2^*) without EIC. Second, explain in which direction the consumer's labor supply is moving when EIC is introduced — which may depend on the location of (x_1^*, x_2^*).

(c) Utility representation $u(x_1, x_2) = \max\{cx_1, x_2\}$ for $(x_1, x_2) \in \mathbb{R}^2_+$ where $c > 0$.

(d) Utility representation $u(x_1, x_2) = 2d \cdot \sqrt{x_1} + x_2$ for $(x_1, x_2) \in \mathbb{R}^2_+$ where $0 < d < 220$.

Notice that (c) and (d) may have several subcases, depending on c and d, respectively.

Department of Economics and the Department of Agricultural and Applied Economics

Ph.D. Qualifying Exam, March 17, 2016

Part 2: Econometrics and Macroeconomics

Part 2A: Econometrics (3 Questions on 3 pages, 90 minutes)
Part 2B: Macroeconomics (2 Questions on 3 pages, 90 minutes)

Note: The minutes assigned to each question indicate the weight given each question by the graders.

Question 1 (30 min.). **(a)** State and explain the Gauss-Markov theorem in the context of the Linear Regression model, assuming that $\sum_{t=1}^{n}(x_t-\overline{x})^2 \neq 0$ and the probabilistic assumptions (2)-(4) (table 1) hold.

Table 1: Traditional Linear Regression model

$$Y_t = \beta_0 + \beta_1 x_t + \varepsilon_t,\ \ t=1,2,...,n$$

$$\left.\begin{array}{l} (1)\ (\varepsilon_t|X_t{=}x_t) \backsim \mathsf{N}(.,.), \\ (2)\ E(\varepsilon_t|X_t{=}x_t) = 0, \\ (3)\ Var(\varepsilon_t|X_t{=}x_t) = \sigma^2, \\ (4)\ E(\varepsilon_t\varepsilon_s|X_t{=}x_t) = 0,\ \ t>s, \end{array}\right\}\ t, s=1,2,...,n$$

(b) Compare and contrast the specification in table 1 with that of table 2 in terms of (i) comparing their assumptions (1)-(4) vs. [1]-[5] and (ii) assessing their validity using a preliminary data analysis.

Table 2: Normal, Linear Regression Model

Statistical GM: $\quad Y_t = \beta_0 + \beta_1 x_t + u_t,\ \ t\in\mathbb{N}:=(1,2,...,n,...)$

$$\left.\begin{array}{lll} [1] & \text{Normality:} & (Y_t|X_t{=}x_t) \backsim \mathsf{N}(.,.),\ f(y_t|x_t;\boldsymbol{\theta}) = \frac{e^{-\frac{(y_t-\beta_0-\beta_1 x_t)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \\ [2] & \text{Linearity:} & E(Y_t|X_t{=}x_t)) = \beta_0 + \beta_1 x_t, \\ [3] & \text{Homoskedasticity:} & Var(Y_t|X_t{=}x_t) = \sigma^2, \\ [4] & \text{Independence:} & \{(Y_t|X_t{=}x_t),\ t\in\mathbb{N}\} \text{ independent process,} \\ [5] & \text{t-invariance:} & \boldsymbol{\theta} := (\beta_0, \beta_1, \sigma^2) \text{ are } \textit{constant} \text{ over } t, \end{array}\right\}\ t\in\mathbb{N}.$$

$$\beta_0 = [E(Y_t) - \beta_1 E(X_t)] \in \mathbb{R},\ \ \beta_1 = \frac{Cov(Y_t,X_t)}{Var(X_t)} \in \mathbb{R},\ \ \sigma^2 = (Var(Y_t) - \frac{[Cov(Y_t,X_t)]2}{Var(X_t)}) \in \mathbb{R}_+$$

(c) (i) Explain why the formulae for the OLS estimators of (β_0, β_1) coincide with those of the Maximum Likelihood (ML) estimators. (ii) Despite that, "the OLS [under (1)-(3)] and ML[under [1]-[5]] estimators of (β_0, β_1) have different finite sampling distributions and optimal *finite* sample properties". Explain.

(d) Discuss the limitations of the Gauss-Markov theorem for inference purposes and explain why its results are not informative enough to test the hypotheses:

$$H_0\!: \beta_1 = 0 \text{ vs. } H_1\!: \beta_1 \neq 0.$$

Question 2 (30 minutes)

1. In reading an article in the journal entitled *Resources and Marine Economics* (ReME) an author estimates by OLS on page 4 the *demand* for crabs in the United States using 30 annual observations since 1980 and reports the following two models.

$$Q_d = \underset{(3.6)}{200} - \underset{(.08)}{.25}\,P_c + \underset{(.07)}{.10}\,P_l \qquad R^2 = .78 \quad \text{Durbin Watson} = 0.75$$

$$Q_d = \underset{(28.6)}{136} - \underset{(.05)}{.12}\,P_c - \underset{(.01)}{.03}\,P_l + \underset{(.012)}{.02}\,P_s + \underset{(.001)}{.009}\,Y \qquad R^2 = .74 \quad \text{Durbin Watson} = 1.98$$

where Q_d is the quantity of crab consumed in the U.S. in a given year, P_c is the price of crab per pound, P_l is the price of lobster per pound, P_s is the price of shrimp per pound, and Y is the per capita income in the U.S. Standard errors for the parameter estimates are given in parenthesis. The t-critical value for $\alpha = .05$ is 2.04 and for $\alpha = .05$ the Durbin Watson the lower critical value is 1.10 and the upper critical value is 1.80. Based on these results answer the following questions.

a. Test if each price coefficient estimate is significantly different from zero using a t-statistic and discuss. Show work!!

b. Compare these two models from an economic and econometric perspective. Note what may seem reasonable and what may seem odd from an economic and econometric standpoint.

c. Discuss how you would test if the parameter estimates on P_s and Y in the second model are jointly not significantly different from zero.

d. Upon further reading in the paper you find on page 6 where the author estimates the *supply* of crabs in the United States using OLS on the same 30 annual observations since 1980 and reports the following results.

$$Q_s = \underset{(58)}{-120} + \underset{(.042)}{.08}\,P_c - \underset{(.06)}{.13}\,W + \underset{(.07)}{.07}\,P_f \qquad R^2 = .63 \quad \text{Durbin Watson} = .82 \text{ (Use same CVs as above)}$$

where the new variables are W = wage rate per hour and P_f = price of fuel per gallon. Again, give an economic and econometric interpretation of these results. Is there any violation of an OLS assumption that seems possible? If so, based on what evidence? If there is a violation then what problem does this violation cause and how would you correct it?

Question 3 (30 minutes)

"Understanding the usefulness of the fixed effect estimator in policy analysis."

Consider the model below:

$$y_{it} = \boldsymbol{X}_{it}\boldsymbol{\beta} + u_{it} = \boldsymbol{X}_{it}\boldsymbol{\beta} + \mu_i + v_{it}.$$

In may panel data applications, the main point of using panel data is to allow for arbitrary correlation between the unobserved time fixed effect μ_i and $\boldsymbol{X}$. The fixed effect analysis achieves this main purpose explicitly through removing the unobserved μ_i.

a) [5 min] The first assumption to be held for the fixed effect (FE) estimator to achieve consistency is:

   $$(\text{A1}): E(v_{it}|\boldsymbol{X}_i, \mu_i) = 0, t = 1, 2, \dots T, \boldsymbol{X}_i \equiv (\boldsymbol{X}_{i1}, \boldsymbol{X}_{i2}, \dots, \boldsymbol{X}_{iT})$$

   The random effect (RE) estimator has to impose an additional assumption: $(\text{A2}): E(\mu_i|\boldsymbol{X}_i) = 0$.

   Please explain (in words) what relaxing (A2) means and why it implies that FE analysis is more robust than RE analysis.

b) [5 min] There is always a price to pay with any gain. Please describe what "price" comes with this robustness gain for FE, and explain in words the reasons based on question a).

c) [5 min] Please provide one alternative way to preserve time-constant variables in an FE analysis.

d) [10 min] Please show in math that the sufficient condition for achieving FE unbiasedness based on (A1) is:

   $$E[\boldsymbol{X}'_{it}(v_{it} - \bar{v}_{i.})] = 0, t = 1,2, \dots, T.$$

   Where $\bar{v}_{i.} = \frac{\sum_{t=1}^{T} v_{it}}{T}$ (i.e., the over -time mean).

   [Hint: please use a time-demeaned equation to state the condition for unbiasedness to start]

e) [5 min] Now assume that among the $\boldsymbol{X}$ there is a policy variable of interest, w_{it}. According to the condition in question d), please explain why the following statement is correct: "FE is often superior to pooled OLS or RE for applications where participation in a program is determined by pre-program attributes that also affect y_{it}."

Questions on Macroeconomics

[1] The Augmented Solow Model (45 minutes)

(i) Output is given by the Cobb-Douglas production function: $Y_t = K_t^a [B_t L_t]^{1-a}$.

(ii) The work force is growing at the exponential rate $(\partial L_t / \partial t)/L_t \equiv \dot{L}_t / L_t = n$, a constant. B_t is growing at the exponential rate $\dot{B}_t / B_t = x$, a constant.

(iii) The rate of change in the capital stock is $\dot{K}_t = sK_t^a [B_t L_t]^{1-a} - \delta K_t$, in which s is the constant saving rate and δ is the depreciation rate.

Let $k \equiv K/L$.

(a) Determine the steady state growth rates for k, the marginal product of labor (the *MPL*), and the marginal product of capital (the *MPK*). (Show your work. Don't just write the necessary equations, but show how you arrive at any needed equations.)

(b) Suppose at time 0 the economy is in its steady state and there is an increase in s. Determine the time paths of ln(*MPL*) and ln(*MPK*) as the economy converges to its new steady state.

Suppose there is a capital stock externality, so $Y_t = \overline{K}_t^b K_t^a L_t^{1-a}$, with $\overline{K}$ being the aggregate capital stock. Of course, we can write $Y_t = \overline{K}_t^b K_t^a L_t^{1-a} = K_t^{a+b} L_t^{1-a}$. ($a$, b >0, but a+b<1.)

(c) Determine the steady state growth rates of k, the marginal product of labor (the *MPL*), and the marginal product of capital (the *MPK*).

[2] Consumption with an uncertain lifetime (45 minutes)

Consider Consumer A.

A lives from time 0 to time $\overline{T}$, and at time 0 maximizes lifetime utility as given by:

$$V(\overline{T}) \equiv \int_0^{\overline{T}} \ln c_t \partial t .$$

(Note: A's rate of time preference is zero.) At time 0, A is endowed with a warehouse containing an inventory of y_0 units of output. A consumes from this inventory over his lifetime. Let y_t be the inventory of goods contained in the warehouse at time t. Then it is the case that

$$\partial y_t / \partial t \equiv \dot{y}_t = -c_t .$$

(a) A will choose the time path of consumption to maximize $V(\overline{T})$, subject to $\partial y_t / \partial t \equiv \dot{y}_t = -c_t$. Write the Hamiltonian for this problem, denoting the state and control variables. Determine c_0 and the optimal path for consumption.

(b) Graph c_t from time 0 to time $\overline{T}$.

Consider individual B.

Consumer B has an uncertain lifespan in that his lifespan (his time to death) is distributed uniformly over the interval $[0, 2\overline{T}]$. (Of course, this implies that his expected lifespan is $\overline{T}$.) B seeks, at time 0, to maximize his expected lifetime utility.

(c) Explain why B's expected lifetime utility is

$$E(V(T)) = \int_0^{2\overline{T}} (2\overline{T})^{-1} [\int_0^{T} \ln(c_t) \partial t] \partial T .$$

One can show that

$$E(V(T)) = \int_0^{2\overline{T}} (2\overline{T})^{-1} [\int_0^{T} \ln(c_t) \partial t] \partial T = \int_0^{2\overline{T}} (1 - t / 2\overline{T}) \ln(c_t) \partial t .$$

B will select his time path of consumption to maximize $E(V(T))$.

As does *A*, *B* at time 0 is endowed with a warehouse containing an inventory of y_0 units of output. As before, the warehouse's inventory of output evolves as $\partial y_t / \partial t \equiv \dot{y}_t = -c_t$.

(d) *B* will choose the time path of consumption to maximize

$$E(V(T)) = \int_0^{2\overline{T}} (1 - t/2\overline{T}) \ln(c_t) \partial t \text{ , subject to } \dot{y}_t = -c_t .$$

Write the Hamiltonian for this problem and determine c_0 and the optimal path for consumption thereafter

(e) Suppose *B* is alive at time t. What is y_t? At the time of death, *B* may leave behind an unconsumed inventory of goods. What is the expected value of his unconsumed output at death? (Write the equation determining this expected value. You need not do the actual calculation.)

(f) Graph *B*'s c_t from time 0 to time $2\overline{T}$, should *B* live until $2\overline{T}$.

(g) How does *A*'s c_0 compare to *B*'s c_0? Explain the intuition behind this result.

(h) How does the presence of an uncertain lifespan impact the time path of consumption? That is, compare *B*'s time path to *A*'s time path. Explain the intuition behind the difference in the two paths.